# Algebraic Methods- Mark Scheme

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

| All $n \in \mathbb{N}$ mod 4 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| All $n^2 \in \mathbb{N}$ mod 4 | 0 | 1 | 0 | 1 |
| All $n^2+2 \in \mathbb{N}$ mod 4 | 2 | 3 | 2 | 3 |

Hence for all $n$, $n^2+2$ is not divisible by 4.

| Question 10 (i) | Scheme | Marks | AOs |
|---|---|---|---|

**Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up**

(i)

**M1:** Awarded for setting up the proof for either the even or odd numbers.

**A1:** Concludes correctly with a reason why $n^2+2$ cannot be divisible by 4 for either $n$ odd or even.

**dM1:** Awarded for setting up the proof for both even and odd numbers

**A1:** Fully correct proof with valid explanation and conclusion for all $n$

**Example of an algebraic proof**

| | | |
|---|---|---|
| For $n=2m, \quad n^2+2=4m^2+2$ | M1 | 2.1 |
| Concludes that this number is not divisible by 4 (as the explanation is trivial) | A1 | 1.1b |
| For $n=2m+1, \quad n^2+2=(2m+1)^2+2=\ldots$ FYI $\left(4m^2+4m+3\right)$ | dM1 | 2.1 |
| Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2+m\right)+3$ ..........AND states ......hence true for all | A1* | 2.4 |
| | **(4)** | |

**Example of a very similar algebraic proof**

| | | |
|---|---|---|
| For $n=2m, \quad \dfrac{4m^2+2}{4}=m^2+\dfrac{1}{2}$ | M1 | 2.1 |
| Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required) | A1 | 1.1b |
| For $n=2m+1, \quad \dfrac{n^2+2}{4}=\dfrac{4m^2+4m+3}{4}=m^2+m+\dfrac{3}{4}$ | dM1 | 2.1 |
| Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all $n$, $n^2+2$ is not divisible by 4 | A1* | 2.4 |
| | **(4)** | |

**Example of a proof via logic**

| | | |
|---|---|---|
| When $n$ is odd, "odd ×odd" = odd | M1 | 2.1 |
| so $n^2+2$ is odd , so (when $n$ is odd) $n^2+2$ cannot be divisible by 4 | A1 | 1.1b |
| When $n$ is even, it is a multiple of 2, so "even ×even" is a multiple of 4 | dM1 | 2.1 |
| Concludes that when $n$ is even $n^2+2$ cannot be divisible by 4 because $n^2$ is divisible by 4.....AND STATES ........trues for all $n$. | A1* | 2.4 |
| | **(4)** | |

**Example of proof via contradiction**

| | | |
|---|---|---|
| Sets up the contradiction<br>'Assume that $n^2+2$ is divisible by 4 $\Rightarrow n^2+2=4k$ ' | M1 | 2.1 |
| $\Rightarrow n^2=4k-2=2(2k-1)$ and concludes even<br>Note that the M mark (for setting up the contradiction must have been awarded) | A1 | 1.1b |
| States that $n^2$ is even, then $n$ is even and hence $n^2$ is a multiple of 4 | dM1 | 2.1 |
| Explains that if $n^2$ is a multiple of 4<br>then $n^2+2$ cannot be a multiple of 4 and hence divisible by 4<br>Hence there is a contradiction and concludes<br>Hence true for all $n$. | A1* | 2.4 |
| | **(4)** | |

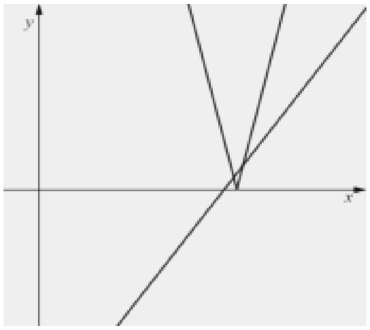
A similar proof exists via contradiction where
A1: $n^2=2(2k-1)\Rightarrow n=\sqrt{2}\times\sqrt{2k-1}$
dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that $n$ is irrational

| **Question 10 (ii)** | **Scheme** | **Marks** | **AOs** |
|---|---|---|---|

**(ii)**
**M1:** States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.
**A1:** and gives an example where it is true,
**Proof using numerical values**

| | | |
|---|---|---|
| SOMETIMES TRUE and chooses any number $x: 9.25 < x < 9.5$ and shows false Eg $x=9.4$ $\lvert 3x-28\rvert=0.2$ and $x-9=0.4$ × | M1 | 2.3 |
| Then chooses a number where it is true Eg $x=12$ $\lvert 3x-28\rvert=8$ $x-9=3$ ✓ | A1 | 2.4 |
| | **(2)** | |

**Graphical Proof**

| | | |
|---|---|---|
|  <br>States or implies ''sometimes true''<br>Sketches both graphs on the same axes.<br>Expect shapes and relative positions to be correct.<br>V shape on +ve $x$ -axis<br>Linear graph with +ve gradient intersecting twice | M1 | 2.3 |

| | | |
|---|---|---|
| Graphs accurate and explains that as there are points where $\|3x-28\| < x-9$ and points where $\|3x-28\| > x-9$ oe in words like 'above' and 'below' or 'dips below at one point' | A1 | 2.4 |
| | **(2)** | |

**Proof via algebra**

| | | |
|---|---|---|
| States sometimes true and attempts to solve<br>both $3x-28 < x-9$ and $-3x+28 < x-9$ or one of these with the bound $9.\dot{3}$ | M1 | 2.3 |
| States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$ | A1 | 2.4 |
| | **(2)** | |

Alt: It is possible to find where it is always true

| | | |
|---|---|---|
| States sometimes true and attempts to solve where it is just true<br>Solves both $3x-28 \geqslant x-9$ and $-3x+28 \geqslant x-9$ | M1 | 2.3 |
| States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$ | A1 | 2.4 |
| | **(2)** | |

May 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

2.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11 (a)** | Attempts $f(4)=2\times 4^3-13\times 4^2+8\times 4+48$ | M1 | 1.1b |
| | $f(4)=0\Rightarrow(x-4)$ is a factor | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | $2x^3-13x^2+8x+48=(x-4)(2x^2...x-12)$ | M1 | 2.1 |
| | $=(x-4)(2x^2-5x-12)$ | A1 | 1.1b |
| | Attempts to factorise quadratic factor or solve quadratic eqn | dM1 | 1.1b |
| | $f(x)=(x-4)^2(2x+3)\Rightarrow f(x)=0$<br>has only two roots, 4 and $-1.5$ | A1 | 2.4 |
| | | **(4)** | |
| **(c)** | Deduces either three roots or deduces that $f(x)$ is moved down two units | M1 | 2.2a |
| | States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the $x$ - axis) | A1 | 2.4 |
| | | **(2)** | |
| **(d)** | For sight of $k=\pm 4,\pm\frac{3}{2}$ | M1 | 1.1b |
| | $k=4,-\frac{3}{2}$ | A1ft | 1.1b |
| | | **(2)** | |
| | | | **(10 marks)** |

**(a)**

**M1:** Attempts to calculate $f(4)$.

Do not accept $f(4)=0$ without sight of embedded values or calculations.

If values are not embedded look for two correct terms from $f(4)=128-208+32+48$

Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection.

See below for awarding these marks.

**A1:** Correct reason with conclusion. Accept $f(4)=0$, hence factor as long as M1 has been scored.

This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $f(4)=0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe.

If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor

**(b)**

**M1:** Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

So for inspection award for $2x^3 - 13x^2 + 8x + 48 = (x-4)\left(2x^2 ... x \pm 12\right)$

$$\begin{array}{r} 2x^2 - 5x \phantom{+ 8x + 48} \\ x-4 \overline{)\, 2x^3 - 13x^2 + 8x + 48} \\ \underline{2x^3 - 8x^2} \phantom{+ 8x + 48} \\ -5x^2 \phantom{+ 8x + 48} \end{array}$$

For division look for

**A1:** Correct quadratic factor $\left(2x^2 - 5x - 12\right)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

**If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $\left(2x^2 - 5x - 12\right)$.**

**dM1:** Correct attempt to solve or factorise their $\left(2x^2 - 5x - 12\right)$ including use of formula

Apply the usual rules $\left(2x^2 - 5x - 12\right) = (ax + b)(cx + d)$ where $ac = \pm 2$ and $bd = \pm 12$

Allow the candidate to move from $(x-4)\left(2x^2 - 5x - 12\right)$ to $(x-4)^2(2x+3)$ for this mark.

**A1:** Via factorisation

Factorises twice to $\mathrm{f}(x) = (x-4)(2x+3)(x-4)$ or $\mathrm{f}(x) = (x-4)^2(2x+3)$ or

$\mathrm{f}(x) = 2(x-4)^2\left(x + \frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x = 4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $f(x) = (x-4)^2(2x+3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorises to $(x-4)(2x^2-5x-12)$ and solves $2x^2-5x-12=0 \Rightarrow x = 4, -\frac{3}{2}$ followed by an explanation that the roots are $4, 4, -\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

**(c)**

**M1:** For a valid **deduction.**

Accept **either** there are 3 roots **or** states that it is a solution of $f(x)=2$ or $f(x)-2=0$

**A1:** Fully explains:

Eg. States three roots, as $f(x)$ is moved down by **two** units (giving three points of intersection with the $x$ - axis)

Eg. States three roots, as it is where $f(x)=2$ (You may see $y=2$ drawn on the diagram)

**(d)**

**M1:** For sight of $\pm 4$ **and** $\pm\frac{3}{2}$ Follow through on ± their roots.

**A1ft:** $k = 4, -\frac{3}{2}$ Follow through on their roots. Accept $4, -\frac{3}{2}$ but not $x = 4, -\frac{3}{2}$

3.

**Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible**

**Generally the marks are awarded for**

**M1**: Suitable approach to answer the question for $n$ being even **OR** odd

**A1:** Acceptable proof for $n$ being even **OR** odd

**M1:** Suitable approach to answer the question for $n$ being even **AND** odd

**A1:** Acceptable proof for $n$ being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of $n$ and then drawing conclusions. So $n=5 \Rightarrow n^3+2=127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8} = \frac{1}{8}n^3 + \frac{1}{4}$ which is not a whole number

- stating $\frac{(n+1)^3+2}{8}=\frac{1}{8}n^3+\frac{3}{8}n^2+\frac{3}{8}n+\frac{3}{8}$ which is not a whole number

**There must be an attempt to generalise either logic or algebra.**

**Example of a logical approach**

| | | | |
|---|---|---|---|
| **Logical approach** | States that if $n$ is odd, $n^3$ is odd | M1 | 2.1 |
| | so $n^3+2$ is odd and therefore cannot be divisible by 8 | A1 | 2.2a |
| | States that if $n$ is even, $n^3$ is a multiple of 8 | M1 | 2.1 |
| | so $n^3+2$ cannot be a multiple of 8<br>So (Given $n \in \mathbb{N}$), $n^3+2$ is not divisible by 8 | A1 | 2.2a |
| | | **(4)** | |
| | | | **4 marks** |

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"$n^3+2$ is odd and cannot be divided by 8 **exactly"**
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so $n^3+2$ is not divisible by 8"
"if $n^3$ is a multiple of 8 then $n^3+2$ cannot be divisible by 8
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

**Example of algebraic approaches**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15**<br>**Algebraic approach** | (If $n$ is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$ | M1 | 2.1 |
| | Eg.'This is 2 more than a multiple of 8, hence not divisible by 8'<br>Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't' | A1 | 2.2a |
| | (If $n$ is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$ | M1 | 2.1 |
| | $=\underline{\underline{8k^3+12k^2+6k+3}}$<br>which is an even number add 3, therefore odd.<br>Hence it is not divisible by 8<br>So (given $n \in \mathbb{N}$,) $n^3+2$ is not divisible by 8 | A1 | 2.2a |
| | | **(4)** | |

| | | | |
|---|---|---|---|
| **Alt algebraic approach** | (If $n$ is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$ | M1 | 2.1 |
| | $=k^3+\frac{1}{4}$ oe<br>which is not a whole number and hence not divisible by 8 | A1 | 2.2a |
| | (If $n$ is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$ | M1 | 2.1 |
| | $=\frac{8k^3+12k^2+6k+3}{8}$ **<br>The numerator is odd as $\underline{\underline{8k^3+12k^2+6k}}+3$ is an even number +3<br>hence not divisible by 8<br>So (Given $n\in\mathbb{N}$,) $n^3+2$ is not divisible by 8 | A1 | 2.2a |
| | | **(4)** | |

Correct expressions are required for the M's. There is no need to state "**If *n* is even,**" $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathbb{N}$

Some students will use $2k-1$ for odd numbers

There is no requirement to change the variable. They may use $2n$ and $2n\pm1$

Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!)

Also **"$=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

## May 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

4.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **9(a)** | $(\mathrm{g}(-2))=4\times-8-12\times4-15\times-2+50$ | M1 | 1.1b |
| | $\mathrm{g}(-2)=0\Rightarrow(x+2)$ is a factor | A1 | 2.4 |
| | | **(2)** | |
| **(b)** | $4x^3-12x^2-15x+50=(x+2)(4x^2-20x+25)$ | M1<br>A1 | 1.1b<br>1.1b |
| | $=(x+2)(2x-5)^2$ | M1<br>A1 | 1.1b<br>1.1b |
| | | **(4)** | |
| **(c)** | (i) $x\leqslant-2,\ x=2.5$ | M1<br>A1ft | 1.1b<br>1.1b |
| | (ii) $x=-1,\ x=1.25$ | B1ft | 2.2a |
| | | **(3)** | |
| | | | **(9 marks)** |

**(a)**

**M1:** Attempts $\mathrm{g}(-2)$ Some sight of ( $-2$ ) embedded or calculation is required.

So expect to see $4\times(-2)^3-12\times(-2)^2-15\times(-2)+50$ embedded

Or $-32-48+30+50$ condoning slips for the M1

Any attempt to divide or factorise is M0. (See demand in question)

**A1:** $\mathrm{g}(-2)=0\Rightarrow(x+2)$ is a factor.

Requires a correct statement and conclusion. Both "$\mathrm{g}(-2)=0$" and "$(x+2)$ is a factor" must be seen in the solution. This may be seen in a preamble before finding $\mathrm{g}(-2)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

**(b)**

**M1:** Attempts to divide $\mathrm{g}(x)$ by $(x+2)$ May be seen and awarded from part (a)

If inspection is used expect to see $4x^3-12x^2-15x+50=(x+2)\left(4x^2\text{...........}\pm25\right)$

If algebraic / long division is used expect to see

$$\begin{array}{r} 4x^2\pm20x \\ x+2\overline{\big)\;4x^3-12x^2-15x+50} \end{array}$$

**A1:** Correct quadratic factor is $\left(4x^2-20x+25\right)$ may be seen and awarded from part (a)

**M1:** Attempts to factorise their $\left(4x^2-20x+25\right)$ usual rule $(ax+b)(cx+d), ac=\pm4, bd=\pm25$

**A1:** $(x+2)(2x-5)^2$ oe seen on a single line. $(x+2)(-2x+5)^2$ is also correct.

Allow recovery for all marks for $\mathrm{g}(x)=(x+2)(x-2.5)^2=(x+2)(2x-5)^2$

**(c)(i)**

**M1:** For identifying that the solution will be where the curve is on or below the axis. Award for either $x\leqslant-2$ or $x=2.5$ Follow through on their $\mathrm{g}(x)=(x+2)(ax+b)^2$ only where $ab<0$ (that is a positive root). Condone $x<-2$ See SC below for $\mathrm{g}(x)=(x+2)(2x+5)^2$

**A1ft:** BOTH $x\leqslant-2$, $x=2.5$ Follow through on their $-\frac{b}{a}$ of their $\mathrm{g}(x)=(x+2)(ax+b)^2$

May see $\{x\leqslant-2\cup x=2.5\}$ which is fine.

(c) (ii)

**B1ft:** For deducing that the solutions of $\mathrm{g}(2x)=0$ will be where $x=-1$ and $x=1.25$

Condone the coordinates appearing $(-1,0)$ and $(1.25,0)$

Follow through on their 1.25 of their $\mathrm{g}(x)=(x+2)(ax+b)^2$

……………………………………………………………………………………………

SC: If a candidate reaches $\mathrm{g}(x)=(x+2)(2x+5)^2$, clearly incorrect because of Figure 2, we will award

In (i) M1 A0 for $x\leqslant-2$ or $x<-2$

In (ii) B1 for $x=-1$ and $x=-1.25$

| | | | |
|---|---|---|---|
| **Alt (b)** | $4x^3-12x^2-15x+50=(x+2)(ax+b)^2$<br>$=a^2x^3+(2ba+2a^2)x^2+(b^2+4ab)x+2b^2$ | | |
| | Compares terms to get either $a$ or $b$ | M1 | 1.1b |
| | Either $a=2$ or $b=-5$ | A1 | 1.1b |
| | Multiplies out expression $(x+2)(\pm2x\pm5)^2$ and compares to $4x^3-12x^2-15x+50$ | M1 | |
| | All terms must be compared or else expression must be multiplied out and establishes that<br>$4x^3-12x^2-15x+50=(x+2)(2x-5)^2$ | A1 | 1.1b |
| | | **(4)** | |

May 2017 Mathematics Advanced Paper 1: Pure Mathematics 2

5.

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** (a) | Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required.<br>$f(-3)=162-63-120+21=0$ so $(x+3)$ is a factor | M1<br>A1<br>**(2)** |
| (b) | Either (Way 1): $f(x)=(x+3)(-6x^2+11x+7)$<br>$=(x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ | M1A1<br>M1A1<br>**(4)** |
| | Or (Way 2) Uses trial or factor theorem to obtain $x=-1/2$ **or** $x=7/3$<br>Uses trial or factor theorem to obtain both $x=-1/2$ **and** $x=7/3$<br>Puts three factors together (see notes below)<br>Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe | M1<br>A1<br>M1<br>A1<br>**(4)** |
| | Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working | M1A1M1A1<br>(4) |
| (c) | $2^y=\frac{7}{3}, \rightarrow \log(2^y)=\log\left(\frac{7}{3}\right)$ or $y=\log_2\left(\frac{7}{3}\right)$ or $\frac{\log(7/3)}{\log 2}$<br>$\{y=1.222392421...\} \Rightarrow y=$ awrt 1.22 | B1, M1<br>A1<br>**(3)**<br>**[9]** |

| | Notes |
|---|---|
| (a) | M1 for attempting either $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ – with **numbers substituted into expression**<br>A1 for calculating $\mathrm{f}(-3)$ **correctly to 0**, and they must state $(x + 3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if f(−3) = 0, (x+3) is a factor".<br>$-6(-3)^3-7(-3)^2+40(-3)+21=0$ so $(x+3)$ is a factor of f(x) is M1A1 providing bracketing is correct. |
| (b) | 1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$.<br>This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b).<br>1st A1: usually for $(-6x^2+11x+7)$ … Credit when seen and use isw if miscopied<br>2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1)<br>2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$<br>but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised.<br>Ignore subsequent work (such as a solution to a quadratic equation.)<br>Way 2: The second M mark needs three roots together so $\pm6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained $\alpha$ and $\beta$ by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0.<br>N.B. Replacing $(-6x^2+11x+7)$ (already awarded M1A1) by $(6x^2-11x-7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0 |
| (c) | B1: $2^y=\frac{7}{3}$<br>M1: Attempt to take logs to solve $2^y=\alpha$ or $2^y=1/\alpha$, where $\alpha>0$ and $\alpha$ was a root of their factorization.<br>A1: for an answer that rounds to 1.22. If other answers are included (and not "rejected") such as ln( – 3) or – 1 lose final A mark<br>**Special case:** Those who deal **throughout** with $\mathrm{f}(x)=6x^3+7x^2-40x-21$<br>They may have full credit in part (a). In part (b) they can achieve a maximum of M1A0M1A0 unless they return the negative sign to give the correct answer. This is then full marks. Part (c) is fine. So they could lose 2 marks on the factorisation. (Like a misread) |

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 2

6.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** | $\mathrm{f}(x)=6x^3+13x^2-4$ | | |
| (a) | $\mathrm{f}\left(-\frac{3}{2}\right)=6\left(-\frac{3}{2}\right)^3+13\left(-\frac{3}{2}\right)^2-4=5$ | Attempting $\mathrm{f}\left(-\frac{3}{2}\right)$ or $\mathrm{f}\left(\frac{3}{2}\right)$ | M1 |
| | | 5 | A1 cao |
| | | | [2] |
| (b) | $\mathrm{f}(-2)=6(-2)^3+13(-2)^2-4$ | Attempts $\mathrm{f}(-2)$. | M1 |
| | $=0$, **and** so $(x+2)$ is a factor. | $\mathrm{f}(-2)=0$ with no sign or substitution errors **and for conclusion.** | A1 |
| | | | [2] |
| (c) | $\mathrm{f}(x)=\{(x+2)\}(6x^2+x-2)$ | | M1 A1 |
| | $=(x+2)(2x-1)(3x+2)$ | | M1 A1 |
| | | | [4] |
| | | | 8 |

| | | Question 4 Notes |
|---|---|---|
| | **Note** | **Long division scores no marks in part (a). The <u>remainder theorem</u> is required.** |
| (a) | **M1** | Attempting $\mathrm{f}\left(-\frac{3}{2}\right)$ or $\mathrm{f}\left(\frac{3}{2}\right)$. $6\left(-\frac{3}{2}\right)^3+13\left(-\frac{3}{2}\right)^2-4$ or $6\left(\frac{3}{2}\right)^3+13\left(\frac{3}{2}\right)^2-4$ is sufficient |
| | **A1** | 5 cao |

| | | |
|---|---|---|
| (b) | **M1** | Attempting $\mathrm{f}(-2)$. (This is **not** given for $\mathrm{f}(2)$) |
| | **A1** | Must correctly show $\mathrm{f}(-2) = 0$ **and** give a conclusion ***in part (b) only***. No simplification of terms is required here. |
| | **Note** | Stating "hence factor" or "it is a factor" or a "tick" or "QED" are possible conclusions.<br>Also a conclusion can be implied from a preamble, eg: "If $\mathrm{f}(-2) = 0$, $(x + 2)$ is a factor…."<br>**Long division scores no marks in part (b). The factor theorem is required.** |
| (c) | **1st M1** | Attempting to divide by $(x + 2)$ leading to a quotient which is quadratic with at least two terms beginning with first term of $\pm 6x^2$ + linear or constant term.<br>Or $\mathrm{f}(x) = (x+2)(\pm 6x^2 + \text{linear and/or constant term})$ (This may be seen in part (b) where candidates did not use factor theorem and might be referred to here) |
| | **1st A1** | $(6x^2 + x - 2)$ seen as quotient or as factor. If there is an error in the division resulting in a remainder give A0, but allow recovery to gain next two marks if $(6x^2 + x - 2)$ is used |
| | **2nd M1**<br>**A1** | For a ***valid*** attempt to factorise **their** three term quadratic.<br>$(x + 2)(2x - 1)(3x + 2)$ and needs all three factors on the same line.<br>Ignore subsequent work (such as a **solution** to a quadratic equation). |
| | **Special cases** | **Calculator methods:**<br>Award M1A1M1A1 for correct answer $(x + 2)(2x - 1)(3x + 2)$ with no working.<br>Award M1A0M1A0 for either $(x + 2)(2x + 1)(3x + 2)$ or $(x + 2)(2x + 1)(3x - 2)$ or $(x + 2)(2x - 1)(3x - 2)$ with no working. (At least one bracket incorrect)<br>Award M1A1M1A1 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x + 2)(2x - 1)(3x + 2)$.<br>Award M0A0M0A0 for a candidate who writes down $x = -2, \frac{1}{2}, -\frac{2}{3}$ giving no factors.<br>Award M1A1M1A1 for $6(x + 2)(x - \frac{1}{2})(x + \frac{2}{3})$ or $2(x + 2)(x - \frac{1}{2})(3x + 2)$ or equivalent<br>Award SC: M1A0M1A0 for $x = -2, \frac{1}{2}, -\frac{2}{3}$ followed by $(x + 2)(x - \frac{1}{2})(x + \frac{2}{3})$. |

May 2015 Mathematics Advanced Paper 1: Pure Mathematics 2

7.

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** | $\mathrm{f}(x) = 6x^3 + 3x^2 + Ax + B$ | |
| **Way 1** (a) | Attempting $\mathrm{f}(1) = 45$ or $\mathrm{f}(-1) = 45$ | M1 |
| | $\mathrm{f}(-1) = -6 + 3 - A + B = 45$ or $-3 - A + B = 45 \Rightarrow B - A = 48$ * (allow $48 = B - A$) | A1 * **cso** |
| | | **(2)** |
| **Way 1 (b)** | Attempting $\mathrm{f}(-\frac{1}{2}) = 0$ | M1 |
| | $6(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 + A(-\frac{1}{2}) + B = 0$ or $-\frac{1}{2}A + B = 0$ or $A = 2B$ | A1 o.e. |
| | Solve to obtain $B = -48$ and $A = -96$ | M1 A1 |
| | | **(4)** |
| **Way 2 (a)** | Long Division<br>$(6x^3 + 3x^2 + Ax + B) \div (x \pm 1) = 6x^2 + px + q$ and sets remainder = 45 | M1 |
| | Quotient is $6x^2 - 3x + (A + 3)$ and remainder is $B - A - 3 = 45$ so $B - A = 48$ * | A1* |
| **Way 2 (b)** | $(6x^3 + 3x^2 + Ax + B) \div (2x + 1) = 3x^2 + px + q$ and sets remainder = 0 | M1 |
| | Quotient is $3x^2 + \frac{A}{2}$ and remainder is $B - \frac{A}{2} = 0$ | A1 |
| | Then Solve to obtain $B = -48$ and $A = -96$ as in scheme above (Way 1) | M1 A1 |

| | | |
|---|---|---|
| **(c)** | Obtain $\left(3x^2-48\right),\left(x^2-16\right),\left(6x^2-96\right),\left(3x^2+\frac{A}{2}\right),\left(3x^2+B\right),\left(x^2+\frac{A}{6}\right)$ or $\left(x^2+\frac{B}{3}\right)$ as factor or as quotient after division by (2$x$ + 1) . Division by ($x$+4) or ($x$-4) see below | B1ft |
| | Factorises $\left(3x^2-48\right),\left(x^2-16\right),\left(48-3x^2\right),\left(16-x^2\right)$or$\left(6x^2-96\right)$ | M1 |
| | = 3 (2$x$ + 1)($x$ + 4)($x$ – 4) (if this answer follows from a wrong $A$ or $B$ then award A0)<br>isw if they go on to solve to give $x$ = 4, -4 and -1/2 | A1cso<br>**(3) [9]** |

Notes

(a) Way 1: **M1:** 1 or –1 substituted into f($x$) and expression put equal to ±45

**A1*:** Answer is given. Must have substituted – 1 and put expression equal to +45. Correct equation with powers of – 1 evaluated and conclusion with no errors seen.

Way 2: **M1**: Long division as far as a remainder which is set equal to ±45

**A1***: See correct quotient and correct remainder and printed answer obtained with no errors

(b) Way 1: **M1**: Must see $\mathrm{f}(-\frac{1}{2})$ and "= 0" unless subsequent work implies this.

**A1**: Give credit for a correct equation **even unsimplified** when first seen, then isw. A correct equation implies M1A1.

**M1**: Attempts to solve the **given equation from part (a)** and their simplified or unsimplified linear equation in $A$ and $B$ from part (b) as far as $A$ =... or $B$ = ...(must eliminate one of the constants but algebra need not be correct for this mark). May just write down the correct answers.

**A1**: **Both** $A$ and $B$ correct

Way 2: **M1**: Long division as far as a remainder which is set equal to 0

**A1**: See correct quotient and correct remainder put equal to 0

**M1A1:** As in Way 1

There may be a mixture of Way 1 for (a) and Way 2 for (b) or vice versa.

(c) **B1**: May be written straight down or from long division, inspection, comparing coefficients or pairing terms

**M1:** Valid attempt to factorise a **listed** quadratic (see general notes) so $(3x-16)(x+3)$ could get M1A0

**A1cso:** (Cannot be awarded if $A$ or $B$ is wrong) Needs the answer in the scheme or -3 (2$x$+1)(4+$x$)(4 – $x$) or equivalent but factor 3 must be shown and there must be all the terms together with brackets.

Way 2: A minority might divide by ($x$- 4) or ($x$ + 4) obtaining $\left(6x^2+27x+12\right)$ or $\left(6x^2-21x-12\right)$ for B1

They then need to factorise $\left(6x^2+27x+12\right)$ or $\left(6x^2-21x-12\right)$ for M1

Then A1cso as before

Special cases:

If they write down f($x$) = 3 (2$x$+1)($x$+4)($x$ - 4) with no working, this is B1 M1 A1

But if they give f($x$) = (2$x$+1)($x$+4)($x$ - 4) with no working (from calculator?) give B1M0A0

And f($x$) = (2$x$ + 1)(3$x$ + 12)($x$ – 4) or f($x$) = (6$x$ + 3)($x$ + 4)($x$ – 4) or f($x$) = (2$x$ + 1)($x$ + 4)(3$x$ – 12) is B1M1A0

May 2014 Mathematics Advanced Paper 1: Pure Mathematics 2

8.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | **If there is no labelling, mark (a) and (b) in that order** | | |
| | $f(x) = 2x^3 - 7x^2 + 4x + 4$ | | |
| **2.**(a) | $f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$ | Attempts $f(2)$ or $f(-2)$ | M1 |
| | $= 0$, and so $(x - 2)$ is a factor. | $f(\mathbf{2}) = 0$ with no sign or substitution errors<br>$(2(2)^3 - 7(2)^2 + 4(2) + 4 = 0$ is sufficient)<br>**and for conclusion.** Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion **but not = 0 just underlined and not hence (2 or f(2)) is a factor**. Note also that a conclusion can be implied from a preamble, eg: "If $f(2) = 0$, $(x - 2)$ is a factor…." | A1 |
| | **Note: Long division scores no marks in part (a). The factor theorem is required.** | | |
| | | | **[2]** |
| (b) | $f(x)= \{(x - 2)\}(2x^2 - 3x - 2)$ | M1: Attempts long division by $(x - 2)$ or other method using $(x - 2)$, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." | M1 A1 |
| | | A1: $(2x^2 - 3x - 2)$ | |
| | $= (x - 2)(x - 2)(2x + 1)$ or $(x - 2)^2(2x + 1)$<br>or equivalent e.g.<br>$= 2(x - 2)(x - 2)(x + \frac{1}{2})$ or $2(x - 2)^2(x + \frac{1}{2})$ | **dM1**: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). **This is dependent** on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the **factors**. | **dM1 A1** |
| | | A1: cao – needs all three factors **on one line**. Ignore following work (such as a solution to a quadratic equation.) | |
| | Note $= (x - 2)(\frac{1}{2}x - 1)(4x + 2)$ would lose the last mark as it is not **fully** factorised | | |
| | **For correct answers only award full marks in (b)** | | |
| | | | **[4]** |
| | | | **Total 6** |

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

9.

| **Question Number** | **Scheme** | **Marks** |
|---|---|---|
| **3. (a)** | Either (Way 1) : Attempt $\text{f}(3)$ or $\text{f}(-3)$<br>$\text{f}(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ *<br>Or (Way 2): Assume $a$ = -9 and attempt $\text{f}(3)$ or $\text{f}(-3)$<br>f(3) = 0 so ($x$ -3) is factor | M1<br>A1 * cso<br>**(2)** |
| | Or (Way 3): $\left(2x^3 - 5x^2 + ax + 18\right) \div (x-3) = 2x^2 + px + q$ where $p$ is a number and $q$ is an expression in terms of $a$<br>Sets the remainder $18+3a+9 = 0$ and solves to give $a = -9$ | M1<br>A1* cso<br>**(2)** |
| **(b)** | Either (Way 1): $\text{f}(x) = (x-3)(2x^2 + x - 6)$<br>$= (x-3)(2x-3)(x+2)$ | M1A1<br>M1A1<br>**(4)** |
| | Or (Way 2) Uses trial or factor theorem to obtain $x$ = -2 **or** $x$ = 3/2<br>Uses trial or factor theorem to obtain both $x$ = -2 **and** $x$ = 3/2<br>Puts three factors together (see notes below)<br>Correct factorisation : $(x-3)(2x-3)(x+2)$ or $(3-x)(3-2x)(x+2)$ or $2(x-3)(x-\frac{3}{2})(x+2)$ oe | M1<br>A1<br>M1<br>A1<br>**(4)** |
| | Or (Way 3) No working three factors $(x-3)(2x-3)(x+2)$ otherwise need working | M1A1M1A1 |
| **(c)** | $\{3^y = 3 \Rightarrow\}$ $\underline{y = 1}$ or $\text{g}(1) = 0$<br>$\{3^y = 1.5 \Rightarrow\} \log\left(3^y\right) = \log 1.5$ or $y = \log_3 1.5$<br>$\{y = 0.3690702...\} \Rightarrow y =$ awrt 0.37 | B1<br>M1<br>A1 **(3)**<br>**[9]** |

| | **Notes for Question 3** |
|---|---|
| **(a)** | M1 for attempting either $\text{f}(3)$ or $\text{f}(-3)$ – with **numbers substituted into expression**<br>A1 for applying $\text{f}(3)$ **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept $\boldsymbol{x}$ = -9) Note that the answer is given in part (a).<br>If they **assume** $a$ = -9 and **verify** by factor theorem or division they must state ($x$ – 3) **is a factor** for A1 (or equivalent such as QED or a tick). |
| **(b)** | 1st M1: attempting to divide by $(x-3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$.<br>(Could divide by $(3-x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc.<br>1st A1: usually for $2x^2 + x - 6 \ldots$ Credit when seen and use isw if miscopied<br>2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1)<br>2nd A1 is cao and needs all three factors together.<br>Ignore subsequent work (such as a solution to a quadratic equation.)<br>NB: $(x-3)(x-\frac{3}{2})(x+2)$ is M1A1M0A0, $(x-3)(x-\frac{3}{2})(2x+4)$ is M1A1M1A0, but $2(x-3)(x-\frac{3}{2})(x+2)$ is M1A1M1A1. |
| **(c)** | B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for g(1) = 0.<br>M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$, but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of f($x$) = 0 (ft their factorization)<br>A1: for an answer that rounds to 0.37. If a third answer is included (and not "rejected") such as ln(-2) lose final A mark |

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

10.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | | | |
| **2.** | | | |
| **(a)** | $f(1)=a+b-4-3=0$ **or** $a+b-7=0$ | Attempt f(±1) | M1 |
| | $a+b=7$ * | Must be f(1) and = **0** needs to be seen | A1 |
| | | | |
| | | | **(2)** |
| **(b)** | $f(-2)=a(-2)^3+b(-2)^2-4(-2)-3=9$ | Attempt f(±2) **and uses** f(±2) = 9 | M1 |
| | $-8a+4b+8-3=9$ | Correct equation with exponents of (-2) removed | A1 |
| | $(-8a+4b=4)$ | | |
| | Solves the **given equation from part (a)** and their equation in *a* and *b* from part (b) as far as *a* =... or *b* = ... | | M1 |
| | $a=2$ **and** $b=5$ | **Both** correct | A1 |
| | Attempts at trial and improvement in (b): Allow the first M1 if they try values for *a* and *b* where *a* + *b* = 7 and substitute their values into the cubic along with *x* = ±2 and sets = 9. For completion to *a* = 2 and *b* = 5 fully shown to be correct allow 4/4. For incomplete or incorrect solutions allow the first M1 only. **If in doubt consult your team leader.** | | |
| | | | **(4)** |
| | | | **[6]** |

| | | |
|---|---|---|
| | **Long Division** | |
| **(a)** | $(ax^3+bx^2-4x-3)\div(x-1)=ax^2+px+q$<br>where $p$ and $q$ are in terms of $a$ or $b$ or both<br>and sets their remainder = 0<br>NB Quotient = $ax^2+(a+b)x+(a+b-4)$ | M1 |
| | a + b = 7 * | A1 |
| | | **(2)** |
| **(b)** | $(ax^3+bx^2-4x-3)\div(x+2)=ax^2+px+q$<br>where $p$ and $q$ are in terms of $a$ or $b$ or both<br>and sets their remainder = 9<br>NB Quotient = $ax^2+(b-2a)x+(4a-4-2b)$ | M1 |
| | 4b – 8a + 5 = 9 | A1 |
| | Follow scheme for final 2 marks | |
| | | |
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| | | |

May 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

11.

| Question number | Scheme | Marks |
|---|---|---|
| **4** (a) | $f(-2)=2.(-2)^3-7.(-2)^2-10.(-2)+24$ | M1 |
| | $=0$ so $(x+2)$ is a factor | A1 |
| | | (2) |
| (b) | $\text{f}(x)=(x+2)(2x^2-11x+12)$ | M1 A1 |
| | $\text{f}(x)=(x+2)(2x-3)(x-4)$ | dM1 A1 |
| | | (4) |
| | | **6 marks** |

| | |
|---|---|
| Notes (a) | **M1** : Attempts f($\pm2$) (Long division is M0)<br>**A1** : is for =0 **and conclusion**<br>**Note**: Stating "hence factor" or "it is a factor" or a "√" (tick) or "QED" is fine for the conclusion.<br>**Note** also that a conclusion can be implied from a <u>preamble</u>, eg: "If $\text{f}(-2)=0$, $(x+2)$ is a factor…." (Not just f(-2)=0) |
| (b) | **1st M1**: Attempts long division by correct factor or other method leading to obtaining $(2x^2\pm ax\pm b)$, $a\neq0$, $b\neq0$, even with a remainder. Working need not be seen as could be done "by inspection."<br>Or *Alternative Method* : **1st M1**: Use $(x+2)(ax^2+bx+c)=2x^3-7x^2-10x+24$ with expansion and comparison of coefficients to obtain $a$ = 2 and to obtain values for $b$ and $c$<br>**1st A1**: For seeing $(2x^2-11x+12)$. [Can be seen here in (b) after work done in (a)]<br>**2nd M1**: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and **needs factors**<br>**2nd A1**: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.)<br>Note: Some candidates will go from $\{(x+2)\}(2x^2-11x+12)$ to $\{x=-2\}$, $x=\frac{3}{2}$, 4, and not list all three factors. Award these responses M1A1M0A0.<br>Finds $x=4$ and $x=1.5$ by factor theorem, formula or calculator and produces factors M1<br>$\text{f}(x)=(x+2)(2x-3)(x-4)$ **or** $\text{f}(x)=2(x+2)(x-1.5)(x-4)$ **o.e. is full marks**<br>$\text{f}(x)=(x+2)(x-1.5)(x-4)$ **loses last A1** |

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

12.

| Question number | Scheme | Marks |
|---|---|---|
| **5 (a)** | $f(-2) = -8 + 4a - 2b + 3 = 7$<br>**so** $2a - b = 6$ * | M1<br>A1<br>(2) |
| **(b)** | $f(1) = 1 + a + b + 3 = 4$<br>Solve two linear equations to give $a = 2$ and $b = -2$ | M1 A1<br>M1 A1<br>(4)<br>**6** |
| Notes | (a) **M1** : Attempts $f(\pm 2) = 7$ or attempts long division as far as putting remainder equal to 7 (There may be sign slips)<br>**A1** is for correct equation with remainder = 7 and for the printed answer with no errors and no wrong working between the two<br>(b) **M1** : Attempts $f(\pm 1) = 4$ or attempts long division as far as putting remainder equal to 4<br>**A1** is for correct equation with remainder = 4 and powers calculated correctly<br>**M1 :** Solving simultaneous equations (may be implied by correct answers). This mark may be awarded for attempts at elimination or substitution leading to values for both a and b. Errors are penalised in the accuracy mark.<br>**A1** is cao for values of $a$ and $b$ and explicit values are needed.<br>Special case: Misreads and puts remainder as 7 again in (b). This may earn M1A0M1A0 in part (b) and will result in a maximum mark of 4/6 | |
| Long Divisions | $\begin{array}{r l} & x^2 + (a-2)x + (b-2a+4) \\ (x+2)\overline{)} & x^3 + ax^2 + bx + 3 \\ & x^3 + 2x^2 \end{array}$ and reach their "$3 - 2b + 4a - 8$" = 7 M1<br>………………………………<br>$\begin{array}{r l} & x^2 + (a+1)x + (b+a+1) \\ (x-1)\overline{)} & x^3 + ax^2 + bx + 3 \\ & x^3 - x^2 \end{array}$ and reach their "$3 + b + a + 1$" = 4 M1<br>……………………<br>A marks as before | |

May 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

13.

| Question Number | Scheme | Marks |
|---|---|---|
| **1.**<br>(a) | $\mathrm{f}(x) = 2x^3 - 7x^2 - 5x + 4$<br>Remainder $= \mathrm{f}(1) = 2 - 7 - 5 + 4 = -6$ — Attempts $\mathrm{f}(1)$ or $\mathrm{f}(-1)$.<br>$= -6$ — $-6$ | <br>M1<br>A1 **[2]** |
| (b) | $\mathrm{f}(-1) = 2(-1)^3 - 7(-1)^2 - 5(-1) + 4$ — Attempts $\mathrm{f}(-1)$.<br>and so $(x + 1)$ is a factor. — $\mathrm{f}(-1) = 0$ with no sign or substitution errors **and for conclusion.** | M1<br>A1 **[2]** |
| (c) | $\mathrm{f}(x) = \{(x + 1)\}(2x^2 - 9x + 4)$<br>$= (x + 1)(2x - 1)(x - 4)$<br>(**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*). | M1 A1<br>dM1 A1<br>**[4]**<br>**8** |
| (a) | M1 for ***attempting*** either $\mathrm{f}(1)$ or $\mathrm{f}(-1)$. Can be implied. Only one slip permitted.<br>M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of $x$. A1 can be given also for $-6$ seen at the bottom of long division working. Award A0 for a candidate who finds $-6$ but then states that the remainder is 6.<br>Award M1A1 for $-6$ without any working. | |
| (b) | M1: attempting only $\mathrm{f}(-1)$. A1: must correctly show $\mathrm{f}(-1) = 0$ and give a conclusion ***in part (b) only***.<br>**Note**: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion.<br>**Note** also that a conclusion can be implied from a preamble, eg: "If $\mathrm{f}(-1) = 0$, $(x + 1)$ is a factor…."<br>**Note: Long division scores no marks in part (b). The factor theorem is required.** | |
| (c) | 1st M1: Attempts long division or other method, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." $(2x^2 \pm ax \pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $\mathrm{f}(x)$ by $(x - 1)$. Eg. Some candidates use their $(2x^2 - 5x - 10)$ in part (c) found from applying a long division method in part (a).<br>1st A1: For seeing $(2x^2 - 9x + 4)$.<br>2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly.<br>2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.)<br>Note: Some candidates will go from $\{(x + 1)\}(2x^2 - 9x + 4)$ to $\{x = -1\}, x = \frac{1}{2}, 4$, and not list all three factors. Award these responses M1A1M1A0.<br>**Alternative:** 1st M1: For finding either $\mathrm{f}(4) = 0$ or $\mathrm{f}\left(\frac{1}{2}\right) = 0$.<br>1st A1: A second correct factor of usually $(x - 4)$ or $(2x - 1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark.<br>2nd dM1: For using two known factors to find the third factor, usually $(2x \pm 1)$.<br>2nd A1 for correct answer of $(x + 1)(2x - 1)(x - 4)$.<br>**Alternative: (for the first two marks)**<br>1st M1: Expands $(x + 1)(2x^2 + ax + b)$ {giving $2x^3 + (a + 2)x^2 + (b + a)x + b$ } then compare coefficients to find values for $a$ and $b$. 1st A1: $a = -9, b = 4$<br>**Not dealing with a factor of 2:** $(x + 1)(x - \frac{1}{2})(x - 4)$ or $(x + 1)(x - \frac{1}{2})(2x - 8)$ scores M1A1M1A0.<br>**Answer only, with one sign error:** eg. $(x + 1)(2x + 1)(x - 4)$ or $(x + 1)(2x - 1)(x + 4)$ scores M1A1M1A0. **(c) Award M1A1M1A1 for Listing all three correct factors with no working.** | |

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

14.

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** <br> **(a)** | $\text{f}(x) = x^4 + x^3 + 2x^2 + ax + b$ | |
| | Attempting $\text{f}(1)$ or $\text{f}(-1)$. | M1 |
| | $\text{f}(1) = 1 + 1 + 2 + a + b = 7$ or $4 + a + b = 7 \Rightarrow a + b = 3$ (as required) **AG** | A1 * cso <br> (2) |
| **(b)** | Attempting $\text{f}(-2)$ or $\text{f}(2)$. | M1 |
| | $\text{f}(-2) = \underline{16 - 8 + 8 - 2a + b = -8}$ $\{\Rightarrow -2a + b = -24\}$ | A1 |
| | Solving both equations simultaneously to get as far as $a = ...$ or $b = ...$ | dM1 |
| | Any one of $a = 9$ or $b = -6$ | A1 |
| | Both $a = 9$ and $b = -6$ | A1 cso <br> (5) <br> **[7]** |

| | **Notes** |
|---|---|
| **(a)** | M1 for attempting either $\text{f}(1)$ or $\text{f}(-1)$. <br> A1 for applying $\text{f}(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a). |
| **(b)** | M1: attempting either $\text{f}(-2)$ or $\text{f}(2)$. <br> A1: correct underlined equation in $a$ and $b$; eg $\underline{16 - 8 + 8 - 2a + b = -8}$ or equivalent, eg $-2a + b = -24$. <br> dM1: an attempt to eliminate one variable from 2 linear simultaneous equations in $a$ and $b$. Note that this mark is dependent upon the award of the first method mark. <br> A1: any one of $a = 9$ or $b = -6$. <br> A1: both $a = 9$ and $b = -6$ and a correct solution only. |
| | **Alternative Method of Long Division:** <br> (a) M1 for long division by $(x - 1)$ to give a remainder in $a$ and $b$ which is independent of $x$. <br> A1 for $\{\text{Remainder} =\}$ $b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given.) <br> (b) M1 for long division by $(x + 2)$ to give a remainder in $a$ and $b$ which is independent of $x$. <br> A1 for $\{\text{Remainder} =\}$ $\underline{b - 2(a - 8) = -8}$ $\{\Rightarrow -2a + b = -24\}$. <br> Then dM1A1A1 are applied in the same way as before. |

Jun 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

15.

| Question Number | Scheme | Marks |
|---|---|---|
| **2** | (a) Attempting to find $\text{f}(3)$ or $\text{f}(-3)$ | M1 |
| | $\text{f}(3) = 3(3)^3 - 5(3)^2 - (58 \times 3) + 40 = 81 - 45 - 174 + 40 = -98$ | A1 (2) |
| | (b) $\{3x^3 - 5x^2 - 58x + 40 = (x-5)\}\ (3x^2 + 10x - 8)$ | M1 A1 |
| | Attempt to <u>factorise</u> 3-term quadratic, or to use the quadratic formula (see general principles at beginning of scheme). This mark may be implied by the correct solutions to the quadratic. | M1 |
| | $(3x-2)(x+4) = 0 \qquad x = \ldots.$ <u>or</u> $x = \dfrac{-10 \pm \sqrt{100+96}}{6}$ | A1 ft |
| | $\dfrac{2}{3}$ (or exact equiv.), $-4$, $5$ (Allow 'implicit' solns, e.g. $\text{f}(5) = 0$, etc.) | A1 (5) |
| | Completely correct solutions without working: full marks. | 7 |

(a) <u>Alternative (long division):</u>

Divide by $(x - 3)$ to get $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1]

$(3x^2 + 4x - 46)$, and $-98$ seen. [A1]

(If continues to say 'remainder = 98', isw)

<u>'Grid' method</u>

$$\begin{array}{r|rrrr} 3 & 3 & -5 & -58 & 40 \\ & 0 & 9 & 12 & -138 \\ \hline & 3 & 4 & -46 & -98 \end{array}$$

(b) 1st M requires use of $(x-5)$ to obtain $(3x^2 + ax + b)$, $a \neq 0, b \neq 0$.

(Working need not be seen… this could be done 'by inspection'.)

<u>'Grid' method</u>

$$\begin{array}{r|rrrr} 3 & 3 & -5 & -58 & 40 \\ & 0 & 15 & 50 & -40 \\ \hline & 3 & 10 & -8 & 0 \end{array}$$

$(3x^2 + 10x - 8) \longleftarrow$

2nd M for the attempt to <u>factorise</u> their 3-term quadratic, or to solve it using the quadratic formula.

Factorisation: $(3x^2 + ax + b) = (3x + c)(x + d)$, where $|cd| = |b|$.

A1ft: Correct factors for their 3-term quadratic <u>followed by a solution</u> (at least one value, which might be incorrect), <u>or</u> numerically correct expression from the quadratic formula for their 3-term quadratic.

<u>Note</u> therefore that if the quadratic is correctly factorised but no solutions are given, the last 2 marks will be lost.

<u>Alternative (first 2 marks):</u>

$(x-5)(3x^2 + ax + b) = 3x^3 + (a-15)x^2 + (b-5a)x - 5b = 0$,

then compare coefficients to find <u>values</u> of $a$ and $b$. [M1]

$a = 10,\ b = -8$ [A1]

<u>Alternative 1</u>: (factor theorem)

M1: Finding that $\text{f}(-4) = 0$

A1: Stating that $(x+4)$ is a factor.

M1: Finding third factor $(x-5)(x+4)(3x \pm 2)$.

A1: Fully correct factors (no ft available here) <u>followed by a solution</u>, (which might be incorrect).

A1: All solutions correct.

<u>Alternative 2</u>: (direct factorisation)

M1: Factors $(x-5)(3x+p)(x+q)$ A1: $pq = -8$

M1: $(x-5)(3x \pm 2)(x \pm 4)$

Final A marks as in Alternative 1.

Throughout this scheme, allow $\left(x \pm \dfrac{2}{3}\right)$ as an alternative to $(3x \pm 2)$.

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

16.

| Question Number | Scheme | Marks |
|---|---|---|
| Q3 (a) | $f\left(\frac{1}{2}\right)=2\times\frac{1}{8}+a\times\frac{1}{4}+b\times\frac{1}{2}-6$ | M1 |
| | $f\left(\frac{1}{2}\right)=-5 \Rightarrow \frac{1}{4}a+\frac{1}{2}b=\frac{3}{4}$ or $a+2b=3$ | A1 |
| | $f(-2)=-16+4a-2b-6$ | M1 |
| | $f(-2)=0 \Rightarrow 4a-2b=22$ | A1 |
| | Eliminating one variable from 2 linear simultaneous equations in $a$ and $b$ | M1 |
| | $a=5$ and $b=-1$ | A1 (6) |
| (b) | $2x^3+5x^2-x-6=(x+2)(2x^2+x-3)$ | M1 |
| | $=(x+2)(2x+3)(x-1)$ | M1A1 (3) |
| | NB $(x+2)(x+\frac{3}{2})(2x-2)$ is A0 But $2(x+2)(x+\frac{3}{2})(x-1)$ is A1 | **[9]** |
| (a) | 1st M1 for attempting $f(\pm\frac{1}{2})$ Treat the omission of the –5 here as a slip and allow the M mark.<br>1st A1 for first correct equation in $a$ and $b$ simplified to three non zero terms (needs –5 used)<br>s.c. If it is not simplified to three terms but is correct and is then used correctly with second equation to give correct answers- this mark can be awarded later.<br>2nd M1 for attempting $f(\mp 2)$<br>2nd A1 for the second correct equation in $a$ and $b$. simplified to three terms (needs 0 used) s.c. If it is not simplified to three terms but is correct and is then used correctly with first equation to give correct answers - this mark can be awarded later.<br>3rd M1 for an attempt to eliminate one variable from 2 linear simultaneous equations in $a$ and $b$<br>3rd A1 for both $a$ = 5 and $b$ = -1 (Correct answers here imply previous two A marks) | |
| (b) | 1st M1 for attempt to divide by ($x$+2) leading to a 3TQ beginning with correct term usually $2x^2$<br>2nd M1 for attempt to factorize their quadratic provided no remainder<br>A1 is cao and needs all three factors<br>Ignore following work (such as a solution to a quadratic equation). | |
| (a) | <u>Alternative;</u><br>M1 for dividing by $(2x-1)$, to get $x^2+(\frac{a+1}{2})x+\text{constant}$ **with remainder as a function of *a* and *b***, and A1 as before for equations stated in scheme .<br>M1 for dividing by $(x+2)$, to get $2x^2+(a-4)x...$ (No need to see remainder as it is zero and comparison of coefficients may be used) with A1 as before | |
| (b) | <u>Alternative;</u><br>M1 for finding second factor correctly by factor theorem, usually ($x$ – 1)<br>M1 for using two known factors to find third factor , usually $(2x\pm 3)$<br>Then A1 for correct factorisation written as product $(x+2)(2x+3)(x-1)$ | |

Jun 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

17.

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | $x^2-9=(x+3)(x-3)$ | B1 |
| | $\frac{4x}{x^2-9}-\frac{2}{(x+3)}=\frac{4x-2(x-3)}{(x+3)(x-3)}$ | M1 |
| | $=\frac{2x+6}{(x+3)(x-3)}$ | A1 |
| | $=\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$ | |
| | $=\frac{2}{(x-3)}$ | A1 |
| | | **(4)** |

B1 $x^2-9=(x+3)(x-3)$ This can occur anywhere.

M1 For combining the two fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

For example accept $\frac{4x}{x^2-9}-\frac{2}{x+3}=\frac{4x(x+3)-2(x^2-9)}{(x+3)(x^2-9)}$

accept separately $\frac{4x}{(x+3)(x-3)}-\frac{2}{(x+3)}=\frac{4x}{(x+3)(x-3)}-\frac{2x-3}{(x+3)(x-3)}$ condoning missing bracket

condone $\frac{4x}{x^2-9}-\frac{2}{x+3}=\frac{4x(x+3)-2}{(x+3)(x^2-9)}$ .........as only one numerator has been adapted

A1 A correct intermediate form of $\frac{\text{simplified linear}}{\text{simplified quadratic}}$

Accept $\frac{2x+6}{(x+3)(x-3)}, \frac{2x+6}{x^2-9}$, and even $\frac{(2x+6)\cancel{(x+3)}}{(x^2-9)\cancel{(x+3)}}$,

A1 Further factorises and cancels (which may be implied) to reach the answer $\frac{2}{x-3}$

Do not penalise correct solutions that include incomplete lines Eg $\frac{4x-2(x-3)}{(x+3)(x-3)}=\frac{4x-2x+6}{\ldots}=\frac{2x+6}{(x+3)(x-3)}=\frac{2}{x-3}$

**This is not a "show that" question.**

**Note:** Watch out for an answer of $\frac{2}{x+3}$ probably scored from $\frac{4x-2(x-3)}{(x+3)(x-3)}=\frac{2x-6}{(x+3)(x-3)}=\frac{2(x-3)}{(x+3)(x-3)}$

This would score B1 M1 A0 A0

June 2012 Mathematics Advanced Paper 1: Pure Mathematics 3

18.

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | $9x^2-4=(3x-2)(3x+2)$ At any stage | B1 |
| | Eliminating the common factor of (3*x*+2) at any stage<br>$\frac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}}=\frac{2}{3x-2}$ | B1 |
| | Use of a common denominator<br>$\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\frac{2(3x+1)}{(3x-2)(3x+1)}-\frac{2(3x-2)}{(3x+1)(3x-2)}$ | M1 |
| | $\frac{6}{(3x-2)(3x+1)}$ or $\frac{6}{9x^2-3x-2}$ | A1 |
| | | **(4 marks)** |

**Notes**

B1 For factorising $9x^2-4=(3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark

B1 For eliminating/cancelling out a factor of (3*x*+2) at any stage of the answer.

M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\frac{2(3x+2)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ Only one numerator adapted, separate fractions

$\frac{2\times3x+1-2\times3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction

A1 $\frac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from 'invisible' brackets.

**Alternative method**

$$\frac{2(3x+2)}{(9x^2-4)}-\frac{2}{(3x+1)}=\frac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)}=\frac{18x+12}{(9x^2-4)(3x+1)}$$ has scored 0,0,1,0 so far

$$=\frac{6\cancel{(3x+2)}}{\cancel{(3x+2)}(3x-2)(3x+1)}$$ is now 1,1,1,0

$$=\frac{6}{(3x-2)(3x+1)}$$ and now 1,1,1,1

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

19.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **2.** | | | |
| **(a)** | $\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$ | | |
| | $= \frac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$ | An attempt to form a single fraction | M1 |
| | $= \frac{8x^2-6x-2}{\{2(x-1)(2x-1)\}}$ | Simplifies to give a correct quadratic numerator over a correct quadratic denominator | A1 aef |
| | $= \frac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ | An attempt to factorise a 3 term quadratic numerator | M1 |
| | $= \frac{4x+1}{2x-1}$ | | A1<br>(4) |
| **(b)** | $\mathrm{f}(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$ | | |
| | $\mathrm{f}(x) = \frac{(4x+1)}{(2x-1)} - 2$ | | |
| | $= \frac{(4x+1)-2(2x-1)}{(2x-1)}$ | An attempt to form a single fraction | M1 |
| | $= \frac{4x+1-4x+2}{(2x-1)}$ | | |
| | $= \frac{3}{(2x-1)}$ | Correct result | A1 *<br>(2) |
| **(c)** | $\mathrm{f}(x) = \frac{3}{(2x-1)} = 3(2x-1)^{-1}$ | | |
| | $\mathrm{f}'(x) = 3(-1)(2x-1)^{-2}(2)$ | | |
| | | $\pm k(2x-1)^{-2}$ | M1 |
| | | | A1 aef |
| | $\mathrm{f}'(2) = \frac{-6}{9} = -\frac{2}{3}$ | Either $\frac{-6}{9}$ or $-\frac{2}{3}$ | A1 |
| | | | (3)<br>**[9]** |

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 3

20.

| Question Number | Scheme | Marks |
|---|---|---|
| **Q1** | $\frac{x+1}{3x^2-3}-\frac{1}{3x+1}$ | |
| | $=\frac{x+1}{3(x^2-1)}-\frac{1}{3x+1}$ | |
| | $=\frac{x+1}{3(x+1)(x-1)}-\frac{1}{3x+1}$<br>$x^2-1\to(x+1)(x-1)$ or<br>$3x^2-3\to(x+1)(3x-3)$ or<br>$3x^2-3\to(3x+3)(x-1)$<br>seen or implied anywhere in candidate's working. | **Award below** |
| | $=\frac{1}{3(x-1)}-\frac{1}{3x+1}$ | |
| | $=\frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$<br>Attempt to combine. | **M1** |
| | or $\frac{3x+1}{3(x-1)(3x+1)}-\frac{3(x-1)}{3(x-1)(3x+1)}$<br>Correct result. | **A1** |
| | ***Decide to award M1 here!!*** | **M1** |
| | $=\frac{4}{3(x-1)(3x+1)}$<br>Either $\frac{4}{3(x-1)(3x+1)}$<br>or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$<br>or $\frac{4}{9x^2-6x-3}$ | **A1 aef** |
| | | **[4]** |