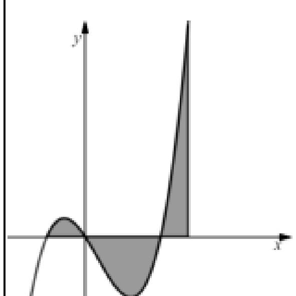
Integration- Mark Scheme

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
8 (a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	1.1b
	$\int x^3 - 2x^2 - 8x \, dx \to \frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$	M1	1.1b
	Attempts area using the correct strategy $\int_{-2}^{0} y \, dx$	dM1	2.2a
	$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0} = (0) - \left(4 - \frac{-16}{3} - 16\right) \quad = \frac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = \pm\frac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	2.2a
	Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$	M1	1.1b
	Achieves $(b+2)^2\left(3b^2 - 20b + 20\right) = 0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x = -2$ and $x = 5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to x^3-2x^2-8x (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function.

The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{0}=\frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1=\frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients.

It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2+\frac{20}{3}=0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4-8b^3-48b^2\pm80=(b+2)(b+2)(3b^2...b...20)$ via repeated division or inspection. FYI $3b^4-8b^3-48b^2+80=(b+2)(3b^3-14b^2-20b+40)$ Allow an attempt via inspection $3b^4-8b^3-48b^2\pm80=(b^2+4b+4)(3b^2...b...20)$ but do not allow candidates to just write out $3b^4-8b^3-48b^2\pm80=(b+2)^2(3b^2-20b+20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2-20b+20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2-20b+20)=0$ with no errors and must have $=0$

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

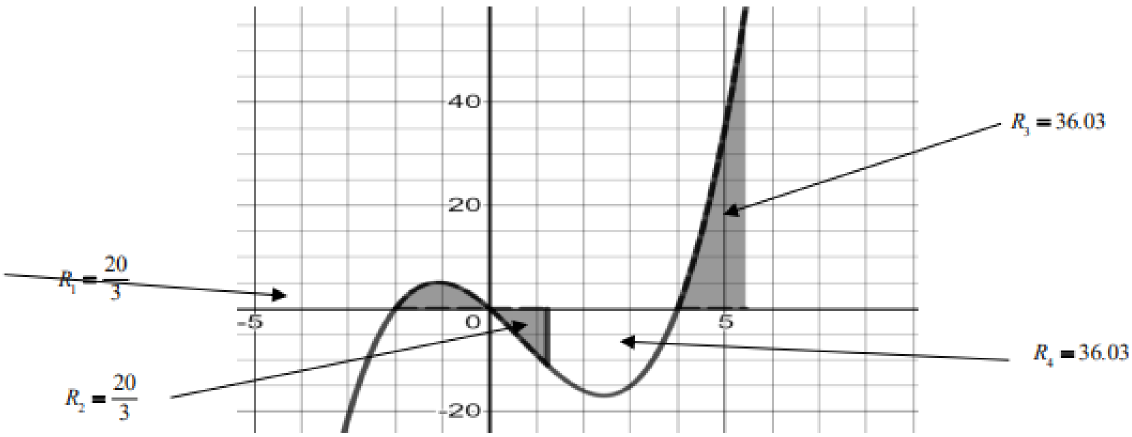
(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 ($x = 5.442$ may not be labelled.)

B1: Explains that (between $x = -2$ and $x = 5.442$) the area above the x-axis = area below the x -axis with appropriate areas shaded or labelled.
Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.
Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



2.

Question	Scheme	Marks	AOs
13 (a)	(i) **Explains** $2x - q = 0$ when $x = 2$ oe Hence $q = 4$ *	B1*	2.4
	(ii) Substitutes $\left(3, \frac{1}{2}\right)$ into $y = \frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2} = \frac{p-9}{(2)\times(6)} \Rightarrow p - 9 = 6 \Rightarrow p = 15$*	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)} - \frac{2.4}{(x+3)}$ oe	A1	1.1b

	$\text{I} = \int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	$\text{I} = \int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}\,dx$ Or $[............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6=\ln 2+\ln 3$ **or** $\ln 8 = 3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $=3.3\ln 6-7.2\ln 2-0.9\ln 2$	dM1	2.1
	$=3.3\ln 3-4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why $q=4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{A}{(2x-4)}+\frac{B}{(x+3)}$ leading to A and B

A1: $\frac{15-3x}{(2x-4)(x+3)}=\frac{1.8}{(2x-4)}-\frac{2.4}{(x+3)}$, or for example $\frac{0.9}{(x-2)}-\frac{2.4}{(x+3)}$, $\frac{9}{(10x-20)}-\frac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area $R = \int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)$

OR $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(x-2)+n\ln(x+3)$

Note that $\int \frac{l}{(x-2)}\,dx \to l\ln(kx-2k)$ and $\int \frac{m}{(x+3)}\,dx \to m\ln(nx+3n)$

A1ft: $= \int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}(dx)$ or $[............]_3^5$ having performed an integral which may be incorrect

dM1: Uses correct ln work seen at least once eg $\ln 6 = \ln 2 + \ln 3$, $\ln 8 = 3\ln 2$ or $m\ln 6k - m\ln 2k = m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $= 3.3\ln 3 - 4.8\ln 2$ oe

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3.

Question	Scheme for Substitution		Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$ Award for • Using a valid substitution $u = \ldots$, changing the terms to u's • integrating and using appropriate limts .		M1	3.1a
	Substitution $u=\sqrt{x+2} \Rightarrow \frac{dx}{du}=2u$ oe	Substitution $u=x+2 \Rightarrow \frac{dx}{du}=1$ oe	B1	1.1b
	$\int 2x\sqrt{x+2}\,dx$ $=\int A\left(u^2 \pm 2\right)u^2 du$	$\int 2x\sqrt{x+2}\,dx$ $=\int A\left(u \pm 2\right)\sqrt{u}du$	M1	1.1b
	$=Pu^5 \pm Qu^3$	$=Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1	2.1
	$=\frac{4}{5}u^5-\frac{8}{3}u^3$	$=\frac{4}{5}u^{\frac{5}{2}}-\frac{8}{3}u^{\frac{3}{2}}$	A1	1.1b
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$ *		A1*	2.1
			(7)	
				(7 marks)

M1: For attempting to integrate using substitution. Look for

- terms and limits changed to u's. Condone slips and errors/omissions on changing $dx \rightarrow du$
- attempted multiplication of terms and raising of at least one power of u by one. Condone slips
- Use of at least the top correct limit. For instance if they go back to x's the limit is 2

B1: For substitution it is for giving the substitution and stating a correct $\frac{dx}{du}$

Eg, $u=\sqrt{x+2} \Rightarrow \frac{dx}{du}=2u$ or equivalent such as $\frac{du}{dx}=\frac{1}{2\sqrt{x+2}}$

M1: It is for attempting to get all aspects of the integral in terms of 'u'.
All terms must be attempted including the dx. You are only condoning slips on signs and coefficients

dM1: It is for using a correct method of expanding and integrating each term (seen at least once) . It is dependent upon the previous M

A1: Correct answer in x or u See scheme

ddM1: Dependent upon the previous M, it is for using the correct limits for their integral, **the correct way around**

A1*: Proceeds correctly to $=\frac{32}{15}\left(2+\sqrt{2}\right)$. **Note that this is a given answer**

There must be at one least correct intermediate line between $\left[\frac{4}{5}u^5-\frac{8}{3}u^3\right]_{\sqrt{2}}^{2}$ and $\frac{32}{15}\left(2+\sqrt{2}\right)$

Question Alt	Scheme for by parts	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,\mathrm{d}x=Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$	M1	1.1b
	$=Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$=\frac{4}{3}x(x+2)^{\frac{3}{2}}-\frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$=\frac{32}{15}\left(2+\sqrt{2}\right)$	A1*	2.1
		(7)	

M1: For attempting using by parts to solve It is a problem- solving mark and all elements do not have to be correct.

- the formula applied the correct way around. You may condone incorrect attempts at integrating $\sqrt{x+2}$ for this problem solving mark
- further integration, again, this may not be correct, and the use of at least the top limit of 2

B1: For $\int\left(\sqrt{x+2}\right)\mathrm{d}x=\frac{2}{3}(x+2)^{\frac{3}{2}}$ oe May be awarded $\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x \to x^2\times\frac{2(x+2)^{\frac{3}{2}}}{3}$

M1: For integration by parts the right way around. Award for $Ax(x+2)^{\frac{3}{2}}-\int B(x+2)^{\frac{3}{2}}(\mathrm{d}x)$

dM1: For integrating a second time. Award for $Ax(x+2)^{\frac{3}{2}}-C(x+2)^{\frac{5}{2}}$

A1: $\frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$ which may be un simplified

ddM1: Dependent upon the previous M, it is for using the limits 2 and 0 the **correct way around**

A1*: Proceeds to $= \frac{32}{15}(2+\sqrt{2})$. **Note that this is a given answer.**

At least one correct intermediate line must be seen. (See substitution). You would condone missing dx's

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4.

Question	Scheme	Marks	AOs
3(a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int\left(\frac{4}{x^3}+kx\right)\mathrm{d}x = -\frac{2}{x^2}+\frac{1}{2}kx^2+c$	A1 A1	1.1b 1.1b
		(3)	
(b)	$\left[-\frac{2}{x^2}+\frac{1}{2}kx^2\right]_{0.5}^{2} = \left(-\frac{2}{2^2}+\frac{1}{2}k\times 4\right)-\left(-\frac{2}{(0.5)^2}+\frac{1}{2}k\times(0.5)^2\right)=8$	M1	1.1b
	$7.5+\frac{15}{8}k=8 \Rightarrow k=...$	dM1	1.1b
	$k=\frac{4}{15}$ oe	A1	1.1b
		(3)	
			(6 marks)

Mark parts (a) and (b) as one

(a)

M1: For $x^n \to x^{n+1}$ for either x^{-3} or x^1. This can be implied by the sight of either x^{-2} or x^2.

Condone " unprocessed" values here. Eg. x^{-3+1} and x^{1+1}

A1: Either term correct (un simplified).

Accept $4\times\frac{x^{-2}}{-2}$ or $k\frac{x^2}{2}$ **with** the indices processed.

A1: Correct (and simplified) with $+c$.

Ignore spurious notation e.g. answer appearing with an $\int$ sign or with dx on the end.

Accept $-\frac{2}{x^2}+\frac{1}{2}kx^2+c$ or exact simplified equivalent such as $-2x^{-2}+k\frac{x^2}{2}+c$

(b)

M1: For substituting both limits into their $-\frac{2}{x^2}+\frac{1}{2}kx^2$, subtracting either way around and setting equal to 8. Allow this when using a changed function. (so the M in part (a) may not have been awarded). Condone missing brackets. Take care here as substituting 2 into the original function gives the same result as the integrated function so you will have to consider both limits.

dM1: For solving a **linear** equation in k. It is dependent upon the previous M only
Don't be too concerned by the mechanics here. Allow for a linear equation in k leading to $k =$

A1: $k=\frac{4}{15}$ or exact equivalent. Allow for $\frac{m}{n}$ where m and n are integers and $\frac{m}{n}=\frac{4}{15}$

Condone the recurring decimal $0.2\dot{6}$ but not 0.266 or 0.267

Please remember to isw after a correct answer

5.

	Scheme	Marks	AOs
13.	The overall method of finding the x coordinate of A.	M1	3.1a
	$y=2x^3-17x^2+40x \Rightarrow \frac{dy}{dx}=6x^2-34x+40$	B1	1.1b
	$\frac{dy}{dx}=0 \Rightarrow 6x^2-34x+40=0 \Rightarrow 2(3x-5)(x-4)=0 \Rightarrow x=...$	M1	1.1b
	Chooses $x=4$ ~~$x=\frac{5}{3}$~~	A1	3.2a
	$\int 2x^3-17x^2+40x\,dx=\left[\frac{1}{2}x^4-\frac{17}{3}x^3+20x^2\right]$	B1	1.1b
	Area $=\frac{1}{2}(4)^4-\frac{17}{3}(4)^3+20(4)^2$	M1	1.1b
	$=\frac{256}{3}$ *	A1*	2.1
		(7)	
			(7 marks)

M1: An overall problem -solving method mark to find the minimum point. To score this you need to see

- an attempt to differentiate with at least **two correct terms**
- an attempt to set their $\frac{dy}{dx}=0$ and then solve to find x. Don't be overly concerned by the mechanics of this solution

B1: $\left(\frac{dy}{dx}=\right)6x^2-34x+40$ which may be unsimplified

M1: Sets their $\frac{dy}{dx}=0$, which must be a 3TQ in x, and attempts to solve via factorisation, formula or calculator. If a calculator is used to find the roots, they must be correct for their quadratic.
If $\frac{dy}{dx}$ is correct allow them to just choose the root 4 for M1 A1. Condone $(x-4)\left(x-\frac{5}{3}\right)$

A1: Chooses $x=4$ This may be awarded from the upper limit in their integral

B1: $\int 2x^3-17x^2+40x\,dx=\left[\frac{1}{2}x^4-\frac{17}{3}x^3+20x^2\right]$ which may be unsimplified

M1: Correct attempt at area. There may be slips on the integration but expect **two correct terms**
The upper limit used must be their larger solution of $\frac{dy}{dx}=0$ and the lower limit used must be 0.
So if their roots are 6 and 10, then they must use 10 and 0. If only one value is found then the limits must be 0 to that value.
Expect to see embedded or calculated values.
Don't accept $\int_0^4 2x^3-17x^2+40x\,dx=\frac{256}{3}$ without seeing the integration and the embedded or calculated values

A1*: Area $=\frac{256}{3}$ **with** correct notation and no errors. Note that this is a given answer.

For correct notation expect to see

- $\frac{dy}{dx}$ or $\frac{d}{dx}$ used correctly at least once. If $f(x)$ is used accept $f'(x)$. Condone y'
- $\int 2x^3-17x^2+40x\,dx$ used correctly at least once with or without the limits.

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6.

Question	Scheme	Marks	AOs
1	$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$		
	Attempts to integrate awarded for any correct power	M1	1.1a
	$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x = \frac{2}{3}\times\frac{x^4}{4} + \ldots + x$	A1	1.1b
	$= \ldots - 6\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \ldots.$	A1	1.1b
	$= \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$	A1	1.1b
			(4 marks)

Notes

M1: Allow for raising power by one. $x^n \to x^{n+1}$
Award for any correct power including sight of $1x$

A1: Correct two **'non fractional power'** terms (may be un-simplified at this stage)

A1: Correct **'fractional power'** term (may be un-simplified at this stage)

A1: Completely correct, simplified and including constant of integration seen on one line.
Simplification is expected for full marks.

Accept correct exact equivalent expressions such as $\frac{x^4}{6} - 4x\sqrt{x} + 1x^1 + c$

Accept $\frac{x^4 - 24x^{\frac{3}{2}} + 6x}{6} + c$

Remember to isw after a correct answer.

Condone poor notation. Eg answer given as $\int \frac{1}{6}x^4 - 4x^{\frac{3}{2}} + x + c$

7.

Question	Scheme	Marks	AOs
15.	For the complete strategy of finding where the normal cuts the x-axis. Key points that must be seen are • Attempt at differentiation • Attempt at using a changed gradient to find equation of normal • Correct attempt to find where normal cuts the x - axis	M1	3.1a
	$y = \frac{32}{x^2} + 3x - 8 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{64}{x^3} + 3$	M1 A1	1.1b 1.1b

	For a correct method of attempting to find Either the equation of the normal: this requires substituting $x=4$ in their $\frac{dy}{dx}=-\frac{64}{x^3}+3=(2)$, then using the perpendicular gradient rule to find the equation of normal $y-6="-\frac{1}{2}"(x-4)$ Or where the equation of the normal at (4,6) cuts the x - axis. As above but may not see equation of normal. Eg $0-6="-\frac{1}{2}"(x-4)\Rightarrow x=...$ or an attempt using just gradients $"-\frac{1}{2}"=\frac{6}{a-4}\Rightarrow a=...$	dM1	2.1
	Normal cuts the x-axis at $x=16$	A1	1.1b
	For the complete strategy of finding the values of the two key areas. Points that must be seen are • There must be an attempt to find the area under the curve by integrating between 2 and 4 • There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-4)\times 6$ or $\int_4^{"16"}"\left(-\frac{1}{2}x+8\right)"dx$ The "16" cannot have just been made up.	M1	3.1a
	$\int \frac{32}{x^2}+3x-8\,dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$	M1 A1	1.1b 1.1b
	Area under curve = $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$	dM1	1.1b
	Total area =10 + 36 =46 *	A1*	2.1
		(10)	
			(10 marks)

(a)

The first 5 marks are for finding the normal to the curve cuts the *x* - axis

M1: For the complete strategy of finding where the normal cuts the x- axis. See scheme

M1: Differentiates with at least one index reduced by one

A1: $\frac{dy}{dx}=-\frac{64}{x^3}+3$

dM1: Method of finding

either the equation of the normal at (4, 6) .
or where the equation of the normal at (4, 6) cuts the x - axis
See scheme. It is dependent upon having gained the M mark for differentiation.

A1: Normal cuts the x-axis at $x=16$

The next 5 marks are for finding the area *R*

M1: For the complete strategy of finding the values of two key areas. See scheme

M1: Integrates $\int \frac{32}{x^2}+3x-8\,dx$ raising the power of at least one index

A1: $\int \frac{32}{x^2}+3x-8\,dx=-\frac{32}{x}+\frac{3}{2}x^2-8x$ which may be unsimplified

dM1: Area $=\left[-\frac{32}{x}+\frac{3}{2}x^2-8x\right]_2^4=(-16)-(-26)=(10)$

It is dependent upon having scored the M mark for integration, for substituting in both 4 and 2 and subtracting either way around. The above line shows the minimum allowed working for a correct answer.

A1*: Shows that the area under curve = 46. No errors or omissions are allowed

....

A number of candidates are equating the line and the curve (or subtracting the line from the curve) The last 5 marks are scored as follows.

M1: For the complete strategy of finding the values of the two key areas. Points that must be seen are

- There must be an attempt to find the area BETWEEN the line and the curve either way around by integrating between 2 and 4
- There must be an attempt to find the area of a triangle using $\frac{1}{2}\times('16'-2)\times\left(-\frac{1}{2}\times2+8\right)$ or via integration $\int_2^{16}\left("-\frac{1}{2}x+8"\right)dx$

M1: Integrates $\int\left("-\frac{1}{2}x+8"\right)-\left(\frac{32}{x^2}+3x-8\right)dx$ either way around and raises the power of at least one index by one

A1: $\pm\left(-\frac{32}{x}+\frac{7}{4}x^2-16x\right)$ must be correct

dM1: Area = $\int_2^4\left("-\frac{1}{2}x+8"\right)-\left(\frac{32}{x^2}+3x-8\right)dx=$ ......either way around

A1: Area = $49-3=46$

NB: Watch for candidates who calculate the area under the curve between 2 and 4 = 10 and subtract this from the large triangle = 56. They will lose both the strategy mark and the answer mark.

NB. Watch for students who use their calculators to do the majority of the work. Please send these items to review

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8.

Question Number	Scheme		Marks
1.	$\int\left(2x^5-\frac{1}{4}x^{-3}-5\right)\mathrm{d}x$		
	Ignore any spurious integral signs throughout		
	$x^n \to x^{n+1}$	Raises any of their powers by 1. E.g. $x^5 \to x^6$ or $x^{-3} \to x^{-2}$ or $k \to kx$ or $x^{\text{their } n} \to x^{\text{their } n+1}$. Allow the powers to be un-simplified e.g. $x^5 \to x^{5+1}$ or $x^{-3} \to x^{-3+1}$ or $kx^0 \to kx^{0+1}$.	M1
	$2\times\frac{x^{5+1}}{6}$ or $-\frac{1}{4}\times\frac{x^{-3+1}}{-2}$	Any one of the first two terms correct **simplified or un-simplified**.	A1
	Two of: $\frac{1}{3}x^6,\ \frac{1}{8}x^{-2},\ -5x$	Any two correct **simplified** terms. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Accept 0.125 for $\frac{1}{8}$ but $\frac{1}{3}$ would clearly need to be identified as 0.3 recurring.	A1
	$\frac{1}{3}x^6+\frac{1}{8}x^{-2}-5x+c$	All correct and simplified and including + c all on one line. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Apply isw here.	A1
			(4 marks)

9.

Question Number	Scheme		Marks
7.(a)	$f'(4)=30+\frac{6-5\times4^2}{\sqrt{4}}$	Attempts to substitutes $x=4$ into $f'(x)=30+\frac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated $f'(x)$	M1
	$f'(4)=-7$	Gradient = -7	A1
	$y-(-8)="-7"\times(x-4)$ or $y="-7"x+c\Rightarrow -8="-7"\times4+c$ $\Rightarrow c=...$	Attempts an equation of a tangent using their numeric $f'(4)$ which has come from substituting $x=4$ into the given $f'(x)$ or their algebraically manipulated $f'(x)$ and $(4,-8)$ with the 4 and -8 correctly placed. If using $y=mx+c$, must reach as far as $c=\ldots$	M1
	$y=-7x+20$	Cao. Allow $y=20-7x$ and allow the "$y=$" to become "detached" but it must be present at some stage. E.g. $y=...$ $=-7x+20$	A1
			(4)
(b)	**Allow the marks in (b) to score in (a) i.e. <u>mark (a) and (b) together</u>**		
	$\Rightarrow f(x)=30x+6\frac{x^{\frac{1}{2}}}{0.5}-5\frac{x^{\frac{5}{2}}}{2.5}(+c)$	M1: $30\rightarrow30x$ or $\frac{6}{\sqrt{x}}\rightarrow\alpha x^{\frac{1}{2}}$ or $-\frac{5x^2}{\sqrt{x}}\rightarrow\beta x^{\frac{5}{2}}$ (these cases only)	M1A1A1
		A1: Any 2 correct terms which can be simplified or un-simplified. This includes the powers – so allow $-\frac{1}{2}+1$ for $\frac{1}{2}$ and allow $\frac{3}{2}+1$ for $\frac{5}{2}$ (With or without $+c$)	
		A1: All 3 terms correct which can be simplified or un-simplified. (With or without $+c$)	
	Ignore any spurious integral signs		

	$x = 4, f(x) = -8 \Rightarrow$ $-8 = 120 + 24 - 64 + c \Rightarrow c = ...$	Substitutes $x = 4, f(x) = -8$ into their f(x) (not f '(x)) i.e. a changed f '(x) containing $+c$ and rearranges to obtain a value or numerical expression for c.	M1
	$\Rightarrow (f(x) =) 30x + 12x^{\frac{1}{2}} - 2x^{\frac{5}{2}} - 88$	Cao and cso (Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$ and e.g. $\sqrt{x^5}$ or $x^2\sqrt{x}$ for $x^{\frac{5}{2}}$). Isw here so as soon as you see the correct answer, award this mark. Note that the "f(x) =" is not needed.	A1
			(5)
			(9 marks)

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10.

Question Number	Scheme	Notes	Marks
1	$\int (2x^4 - \frac{4}{\sqrt{x}} + 3)\mathrm{d}x$		
	$\frac{2}{5}x^5 - \frac{4}{\frac{1}{2}}x^{\frac{1}{2}} + 3x$	M1: $x^n \to x^{n+1}$. One power increased by 1 but not for just + c. This could be for 3 → $3x$ or for $x^n \to x^{n+1}$ on what they think $\frac{1}{\sqrt{x}}$ is as a power of x. A1: One of these 3 terms correct. Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}, -\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}, 3x^1$ A1: Two of these 3 terms correct. Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}, -\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}, 3x^1$	M1A1A1
	$= \frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$	**Complete fully correct simplified expression appearing all on one line with constant.** Allow 0.4 for $\frac{2}{5}$. Do not allow $3x^1$ for $3x$ Allow $\sqrt{x}$ or $x^{0.5}$ for $x^{\frac{1}{2}}$	A1
	Ignore any spurious integral signs and ignore subsequent working following a fully correct answer.		
			[4]
			4 marks

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11.

Question Number	Scheme		Marks
3.	$y = 4x^3 - \frac{5}{x^2}$		
(a)	$12x^2 + \frac{10}{x^3}$	M1: $x^n \to x^{n-1}$ e.g. Sight of x^2 or x^{-3} or $\frac{1}{x^3}$	M1A1A1
		A1: $3 \times 4x^2$ or $-5 \times -2x^{-3}$ (oe) (Ignore + c for this mark)	
		A1: $12x^2 + \frac{10}{x^3}$ or $12x^2 + 10x^{-3}$ all on one line and no + c	
	Apply ISW here and award marks when first seen.		
			(3)
(b)	$x^4 + \frac{5}{x} + c$ or $x^4 + 5x^{-1} + c$	M1: $x^n \to x^{n+1}$. e.g. Sight of x^4 or x^{-1} or $\frac{1}{x^1}$ **Do not award for integrating their answer to part (a)**	M1A1A1
		A1: $4\frac{x^4}{4}$ or $-5 \times \frac{x^{-1}}{-1}$	
		A1: For fully correct and simplified answer with + c all on one line. Allow $x^4 + 5 \times \frac{1}{x} + c$ Allow $1x^4$ for x^4	
	Apply ISW here and award marks when first seen. Ignore spurious integral signs for all marks.		
			(3)
			(6 marks)

12.

Question Number	Scheme		Marks
10(a)	$f(x)=x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x(+c)$	M1: $x^n \to x^{n+1}$ A1: Two terms in x correct, simplification is not required in coefficients or powers A1: All terms in x correct. Simplification not required in coefficients or powers and $+$ c is not required	M1A1A1
	Sub $x=4, y=9$ into $f(x) \Rightarrow c=...$	M1: Sub $x = 4$, $y = 9$ into f (x) to obtain a value for c. If no $+ c$ then M0. Use of $x = 9$, $y = 4$ is M0.	M1
	$(f(x)=)x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x+2$	Accept equivalents **but must be simplified** e.g. $f(x)=x^{\frac{3}{2}}-4.5\sqrt{x}+2x+2$ Must be all 'on one line' **and simplified.** Allow $x\sqrt{x}$ for $x^{\frac{3}{2}}$	A1
			(5)
(b)	Gradient of normal is $-\frac{1}{2}\Rightarrow$ Gradient of tangent $= +2$	M1: Gradient of $2y+x=0$ is $\pm\frac{1}{2}(m)\Rightarrow\frac{dy}{dx}=-\frac{1}{\pm\frac{1}{2}}$ A1: Gradient of tangent = +2 (May be implied)	M1A1
	The A1 may be implied by $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2}=-\frac{1}{2}$		
	$\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2=2\Rightarrow\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}=0$	Sets the given $f'(x)$ **or their** $f'(x)$ = their **changed** m and **not** their m where m has come from $2y+x=0$	M1
	$\times 4\sqrt{x}\Rightarrow 6x-9=0\Rightarrow x=..$	$\times 4\sqrt{x}$ or equivalent **correct algebraic** processing (**allow sign/arithmetic errors only**) and attempt to solve to obtain a value for x. If $f'(x)\neq 2$ they need to be solving a three term quadratic in $\sqrt{x}$ correctly and square to obtain a value for x. **Must be using the given** $f'(x)$ **for this mark.**	M1

	$x = 1.5$	$x = \frac{3}{2}(1.5)$ Accept equivalents e.g . $x = \frac{9}{6}$ **If any 'extra' values are not rejected, score A0.**	A1
			(5)
	Beware $\frac{-1}{\frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2} = -\frac{1}{2} \Rightarrow \frac{-2}{3\sqrt{x}} + \frac{4\sqrt{x}}{9} - \frac{1}{2} = -\frac{1}{2}$ etc. leads to the correct answer and could score M1A1M1M0(incorrect processing)A0		
			(10 marks)

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13.

Question Number	Scheme	Marks
1.	$\int (8x^3 + 4)\,dx = \frac{8x^4}{4} + 4x$	M1, A1
	$= 2x^4 + 4x + c$	A1
		(3 marks)

Notes

M1 $x^n \to x^{n+1}$ so $x^3 \to x^4$ or $4 \to 4x$ or $4x^1$

A1 This is for either term with coefficient unsimplified (power must be simplified)– so $\frac{8}{4}x^4$ or $4x$ (accept $4x^1$)

A1 Fully correct simplified solution with c i.e. $2x^4 + 4x + c$ [allow $2x^4 + 4x + cx^0$]

If the answer is given as $\int 2x^4 + 4x + c$, with an integral sign – having never been seen as the fully correct simplified answer without an integral sign – then give M1A1A0 but allow anything before the = sign e.g. $y = 2x^4 + 4x + c$, $f(x) = 2x^4 + 4x + c$, $\int = 2x^4 + 4x + c$, etc.…

If this answer is followed by (for example) $x^4 + 2x + k$ then treat this as **isw** (ignore subsequent work)
If they follow it by finding a value for c, also **isw**, provided correct answer with c has been seen and credited

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14.

Question Number	Scheme		Marks
2	$(\int =) \frac{10x^5}{5} - \frac{4x^2}{2}, - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}}$	M1: Some attempt to integrate: $x^n \to x^{n+1}$ on at least one term. (not for + c) (If they think $\frac{3}{\sqrt{x}}$ is $3x^{\frac{1}{2}}$ you can still award the method mark for $x^{\frac{1}{2}} \to x^{\frac{3}{2}}$	M1A1, A1
		A1: $\frac{10x^5}{5}$ **and** $\frac{-4x^2}{2}$ or better	
		A1: $-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}$ or better	
	$= \underline{2x^5 - 2x^2 - 6x^{\frac{1}{2}} + c}$	Each term correct and simplified and the + *c* all appearing together on the same line. Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$. Ignore any spurious integral or signs and/or d*y*/d*x*'s.	A1
	Do **not** apply isw. If they obtain the correct answer and then e.g. divide by 2 they lose the last mark.		
			[4]

15.

Question Number	Scheme		Marks
9 (a)	$(3-x^2)^2 = 9 - 6x^2 + x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4, \quad p, q \neq 0$	M1
	$9x^{-2} + x^2$	Must come from $\frac{9 + x^4}{x^2}$	A1
	-6	Must come from $\frac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\frac{(3-x^2)^2}{x^2}$ as $(3x^{-1} - x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9 + Ax^2 + Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)

	$(f'(x) = 9x^{-2} - 6 + x^2)$		
(b)	$-18x^{-3} + 2x$	M1: $x^n \rightarrow x^{n-1}$ on separate terms at least once. Do not award for $A \rightarrow 0$ (Integrating is M0) A1ft: $-18x^{-3} + 2"B"x$ with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)

(c)	$f(x) = -9x^{-1} - 6x + \frac{x^3}{3} (+c)$	M1: $x^n \rightarrow x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1} + Ax + \frac{Bx^3}{3} (+c)$ with numerical A and B, $A, B \neq 0$	M1A1ft
	$10 = \frac{-9}{-3} - 6(-3) + \frac{(-3)^3}{3} + c$ so c = ...	Uses x = -3 and y =10 in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in c and attempts to find c. No + c gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	c = -2	cso	A1
	$(f(x) =) - 9x^{-1} - 6x + \frac{x^3}{3}$ + their c	Follow through their c in an otherwise (possibly un-simplified) **correct expression**. Allow $-\frac{9}{x}$ for $-9x^{-1}$ or even $\frac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

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16.

Question Number	Scheme		Marks
8.	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) \quad -x^3 + "2"x^{-2} - "\left(\frac{5}{2}\right)"x^{-3}$		M1
	$(y=) \quad -\frac{1}{4}x^4 + \frac{"2"x^{-1}}{(-1)} - "\left(\frac{5}{2}\right)"\frac{x^{-2}}{(-2)} (+c)$	Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
	$(y=) \quad -\frac{1}{4}x^4 + \frac{2x^{-1}}{(-1)} - \frac{5}{2}\frac{x^{-2}}{(-2)} (+c)$	This is not follow through – must be correct	A1
	Given that $y = 7$, at $x = 1$, then $7 = -\frac{1}{4} - 2 + \frac{5}{4} + c \Rightarrow c =$		M1
	So, $(y =) \quad -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + c, \; c = 8$ **or** $(y=) -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + 8$		A1
			[6]
			6 marks

	Notes
	M1: Expresses as three term polynomial with powers 3, -2 and –3. Allow slips in coefficients. This may be implied by later integration having all three powers 4, -1 and -2. **M1:** An attempt to integrate at least one term so $x^n \to x^{n+1}$ (not a term in the numerator or denominator) **A1ft:** Any two integrations are correct – coefficients may be unsimplified (follow through errors in **coefficients only here**) so should have two of the powers 4, -1 and –2 after integration – depends on 2nd method mark only. There should be a maximum of three terms here. **A1:** Correct three terms – coefficients **may be unsimplified**- do not need constant for this mark Depends on both Method marks **M1: Need constant for this mark.** Uses $y = 7$ and $x = 1$ in their changed expression in order to find c, and attempt to find c. *This mark is available even after there is suggestion of differentiation.* **A1:** Need **all four** correct terms to be **simplified** and need $c = 8$ here.

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17.

Question Number	Scheme	Marks
1.	$\left\{\int\left(6x^2 + \frac{2}{x^2} + 5\right)\mathrm{d}x\right\} = \frac{6x^3}{3} + \frac{2x^{-1}}{-1} + 5x\,(+c)$	M1 A1
	$= 2x^3 - 2x^{-1}\,;\, + 5x + c$	A1; A1
		4

	Notes	
	M1: for some attempt to integrate a term in x: $x^n \to x^{n+1}$ So seeing either $6x^2 \to \pm\lambda x^3$ or $\frac{2}{x^2} \to \pm\mu x^{-1}$ or $5 \to 5x$ is M1. **1st A1**: for a correct un-simplified x^3 or x^{-1} $\left(\text{or } \frac{1}{x}\right)$ term. **2nd A1:** for both x^3 and x^{-1} terms correct and simplified on the same line. Ie. $2x^3 - 2x^{-1}$ or $2x^3 - \frac{2}{x}$. **3rd A1:** for $+5x + c$. Also allow $+5x^1 + c$. This needs to be written on the same line. **Ignore the incorrect use of the integral sign in candidates' responses.** **Note:** If a candidate scores M1A1A1A1 and their answer is NOT ON THE SAME LINE then withhold the final accuracy mark.	

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18.

Question	Scheme	Marks
1. **(a)**	$4x^3 + 3x^{-\frac{1}{2}}$	M1A1A1 (3)
(b)	$\frac{x^5}{5} + 4x^{\frac{3}{2}} + C$	M1A1A1 (3)
		6 marks
	Notes	
(a)	M1 for $x^n \to x^{n-1}$ i.e. x^3 or $x^{-\frac{1}{2}}$ seen 1st A1 for $4x^3$ <u>or</u> $6\times\frac{1}{2}\times x^{-\frac{1}{2}}$ (o.e.) (ignore any $+c$ for this mark) 2nd A1 for simplified terms i.e. <u>both</u> $4x^3$ <u>and</u> $3x^{-\frac{1}{2}}$ or $\frac{3}{\sqrt{x}}$ and no $+c$ $\left[\frac{3}{1}x^{-\frac{1}{2}} \text{ is A0}\right]$ Apply ISW here and award marks when first seen	
(b)	M1 for $x^n \to x^{n+1}$ applied to y only so x^5 or $x^{\frac{3}{2}}$ seen. Do not award for integrating their answer to part (a) 1st A1 for $\frac{x^5}{5}$ or $\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ (or better). Allow $1/5x^5$ here but not for 2nd A1 2nd A1 for fully correct and simplified answer with $+C$. Allow $(1/5)x^5$ If $+C$ appears earlier but not on a line where 2nd A1 could be scored then A0	

19.

Question	Scheme	Marks
7.	$[\mathrm{f}(x)=]\frac{3x^3}{3}-\frac{3x^2}{2}+5x[+c]$ <u>or</u> $\left\{x^3-\frac{3}{2}x^2+5x(+c)\right\}$	M1A1
	$10=8-6+10+c$	M1
	$c=-2$	A1
	$\mathrm{f}(1)=1-\frac{3}{2}+5$ "-2" $=\underline{\frac{5}{2}}$ (o.e.)	A1ft (5)
		5 marks
	Notes	
	1^{st} M1 for attempt to integrate $x^n \to x^{n+1}$ 1^{st} A1 all correct, possibly unsimplified. Ignore $+c$ here. 2^{nd} M1 for using $x=2$ <u>and</u> f(2) = 10 to form a linear equation in c. Allow sign errors. They should be substituting into a <u>changed</u> expression 2^{nd} A1 for $c=-2$ 3^{rd} A1ft for $\frac{9}{2}+c$ Follow through their <u>numerical</u> c ($\neq 0$) This mark is dependent on 1^{st} M1 and 1^{st} A1 only.	

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20.

Question Number	Scheme	Marks
2. **(a)**	$\frac{\mathrm{d}y}{\mathrm{d}x}=10x^4-3x^{-4}$ or $10x^4-\frac{3}{x^4}$	M1 A1 A1 (3)
(b)	$\left(\int=\right)\frac{2x^6}{6}+7x+\frac{x^{-2}}{-2}=\frac{x^6}{3}+7x-\frac{x^{-2}}{2}$	M1 A1 A1
	$+C$	B1 (4)
		7

	Notes	
	(a) M1: Attempt to differentiate $x^n \to x^{n-1}$ (for any of the 3 terms) i.e. ax^4 or ax^{-4}, where a is any non-zero constant or the 7 differentiated to give 0 is sufficient evidence for M1 1st A1: One correct (non-zero) term, possibly unsimplified. 2nd A1: Fully correct **simplified** answer. (b) M1: Attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant). 1st A1: Two correct terms, possibly unsimplified. 2nd A1: All three terms correct and **simplified**. Allow correct equivalents to printed answer , e.g. $\frac{x^6}{3}+7x-\frac{1}{2x^2}$ or $\frac{1}{3}x^6+7x-\frac{1}{2}x^{-2}$ Allow $\frac{1x^6}{3}$ or $7x^1$ B1: + C appearing at any stage in part (b) (independent of previous work)	

21.

Question Number	Scheme	Marks
6. **(a)**	$p=\frac{1}{2},\ q=2$ or $6x^{\frac{1}{2}},\ 3x^2$	B1, B1 (2)
(b)	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{3x^3}{3}\quad \left(=4x^{\frac{3}{2}}+x^3\right)$ $x=4, y=90$: $32+64+C=90 \Rightarrow C=-6$ $y=4x^{\frac{3}{2}}+x^3+\text{"}their-6\text{"}$	M1 A1ft M1 A1 A1 (5) **7**

	Notes	
	(a) Accept any equivalent answers, e.g. $p = 0.5$, $q = 4/2$ (b) 1st M: Attempt to integrate $x^n \rightarrow x^{n+1}$ (for either term) 1st A: ft their p and q, but terms need not be simplified ($+C$ not required for this mark) 2nd M: Using $x = 4$ <u>and</u> $y = 90$ to form an equation in C. 2nd A: cao 3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Numerator and denominator integrated separately</u>: First M mark **cannot** be awarded so only mark available is second M mark. So 1 out of 5 marks.	

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22.

Question Number	Scheme	Marks
2.	$\left(\int =\right)\dfrac{12x^6}{6}, -\dfrac{3x^3}{3}, +\dfrac{4x^{\frac{4}{3}}}{\frac{4}{3}}, (+c)$ $= \underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}$	**M1A1,A1,A1** **A1** **5**
	Notes	
	M1 for some attempt to integrate: $x^n \to x^{n+1}$ i.e ax^6 or ax^3 or $ax^{\frac{4}{3}}$ or $ax^{1\frac{1}{3}}$, where a is a non zero constant 1st A1 for $\dfrac{12x^6}{6}$ or better 2nd A1 for $-\dfrac{3x^3}{3}$ or better 3rd A1 for $\dfrac{4x^{\frac{4}{3}}}{\frac{4}{3}}$ or better 4th A1 for each term correct and simplified and the $+c$ occurring in the final answer	

23.

Question Number	Scheme	Marks
7.	$\left(\text{f}(x)=\right)\dfrac{12x^3}{3} - \dfrac{8x^2}{2} + x(+c)$ $\left(\text{f}(-1)=0 \Rightarrow\right)\ 0 = 4\times(-1) - 4\times 1 - 1 + c$ $c = \underline{9}$ $\left[\text{f}(x) = 4x^3 - 4x^2 + x + 9\right]$	**M1 A1 A1** **M1** **A1** **5**
	Notes	
	1st M1 for an attempt to integrate $x^n \to x^{n+1}$ 1st A1 for at least 2 terms in x correct - needn't be simplified, ignore $+c$ 2nd A1 for all the terms in x correct but they need not be simplified. No need for $+c$ 2nd M1 for using $x = -1$ and $y = 0$ to form a linear equation in c. No $+c$ gets M0A0 3rd A1 for $c = 9$. Final form of f(x) is not required.	

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24.

Question Number	Scheme	Marks
2.	$\frac{8x^4}{4}+\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}-5x+c$	M1 A1
	$= 2x^4+4x^{\frac{3}{2}}, -5x+c$	A1 A1
		4
	Notes	
	M1 for some attempt to integrate a term in x: $x^n \to x^{n+1}$ 1st A1 for correct, possibly un-simplified x^4 or $x^{\frac{3}{2}}$ term. e.g. $\frac{8x^4}{4}$ or $\frac{6x^{\frac{3}{2}}}{\frac{3}{2}}$ 2nd A1 for <u>both</u> $2x^4$ and $4x^{\frac{3}{2}}$ terms correct and simplified on the same line N.B. some candidates write $4\sqrt{x^3}$ or $4x^{1\frac{1}{2}}$ which are, of course, fine for A1 3rd A1 for $-5x+c$. Accept $-5x^1+c$. The $+c$ must appear on the same line as the $-5x$ N.B. We do not need to see one line with a fully correct integral Ignore ISW (ignore incorrect subsequent working) if a correct answer is followed by an incorrect version. Condone poor use of notation e.g. $\int 2x^4+4x^{\frac{3}{2}}-5x+c$ will score full marks.	

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25.

Question number	Scheme	Marks
Q4	$x\sqrt{x} = x^{\frac{3}{2}}$ (Seen, or implied by correct integration)	B1
	$x^{-1/2} \to kx^{1/2}$ or $x^{3/2} \to kx^{5/2}$ (k a non-zero constant)	M1
	$(y =)\ \dfrac{5x^{1/2}}{1/2}\ldots + \dfrac{x^{5/2}}{5/2}$ $(+C)$ ("$y =$" and " $+C$" are not required for these marks)	A1… A1
	$35 = \dfrac{5 \times 4^{1/2}}{1/2} + \dfrac{4^{5/2}}{5/2} + C$ An equation in C is required (see conditions below). (With their terms simplified or unsimplified).	M1
	$C = \dfrac{11}{5}$ or equivalent $2\dfrac{1}{5}$, 2.2	A1
	$y = 10x^{1/2} + \dfrac{2x^{5/2}}{5} + \dfrac{11}{5}$ (Or equivalent simplified)	A1 ft
	I.s.w. if necessary, e.g. $y = 10x^{1/2} + \dfrac{2x^{5/2}}{5} + \dfrac{11}{5} = 50x^{1/2} + 2x^{5/2} + 11$ The final A mark requires an equation "$y = \ldots$" with correct x terms (see below).	[7]

	B mark: $x^{\frac{3}{2}}$ often appears from integration of $\sqrt{x}$, which is B0.	
	1st A: Any unsimplified or simplified correct form, e.g. $\dfrac{5\sqrt{x}}{0.5}$.	
	2nd A: Any unsimplified or simplified correct form, e.g. $\dfrac{x^2\sqrt{x}}{2.5}$, $\dfrac{2(\sqrt{x})^5}{5}$.	
	2nd M: Attempting to use $x = 4$ and $y = 35$ in a changed function (even if differentiated) to form an equation in C.	
	3rd A: Obtaining $C = \dfrac{11}{5}$ with no earlier incorrect work.	
	4th A: Follow-through only the value of C (i.e. the other terms must be correct). Accept equivalent simplified terms such as $10\sqrt{x} + 0.4x^2\sqrt{x}$ ...	

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26.

Question Number	Scheme	Marks
6. (a)	Seeing -4 and 2.	B1 **(1)**
(b)	$x(x+4)(x-2) = \underline{x^3 + 2x^2 - 8x}$ or $\underline{x^3 - 2x^2 + 4x^2 - 8x}$ (without simplifying)	B1
	$\int (x^3 + 2x^2 - 8x)\,dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \{+c\}$ or $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2} \{+c\}$	M1A1ft
	$\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{0} = (0) - \left(64 - \frac{128}{3} - 64\right)$ **or** $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{0}^{2} = \left(4 + \frac{16}{3} - 16\right) - (0)$	dM1
	One integral $= \pm 42\frac{2}{3}$ (42.6 or awrt 42.7) **or** other integral $= \pm 6\frac{2}{3}$ (6.6 or awrt 6.7)	A1
	Hence Area $=$ "$their\,42\frac{2}{3}$" $+$ "$their\,6\frac{2}{3}$" or Area $=$ "$their\,42\frac{2}{3}$" $-$ "$-\,their\,6\frac{2}{3}$"	dM1
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	A1
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	**(7)**
		[8]

	Notes for Question 6
(a)	B1: Need both –4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or A = -4, B = 2 or indeed any indication of -4 and 2 – check graph also
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER -a and 0 and subtracting either way round OR similarly for 0 and b. **If their limits –a and b are used in ONE integral, apply the Special Case below.** A1: Obtain **either** $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)*from the integral from -4 to 0* **or** $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ **or** $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) **Correct** method **to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from** two separate **definite integrals**. A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen. For full marks on this question there must be **two** definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. **(Trapezium rule gets no marks after first two B marks)**
(b)	**Special Case: one integral only from –a to b:** B1M1A1 available as before, then $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{2} = (4 + \frac{16}{3} - 16) - \left(64 - \frac{128}{3} - 64\right) = -6\frac{2}{3} + 42\frac{2}{3} = ...$ **dM1** for correct use of their limits –a and b and **subtracting** either way round. A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)

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27.

<table>
<tr><th>Question number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>Method 1
5 (a)</td><td>Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic</td><td>Or puts $y = 10(10-y) - (10-y)^2 - 8$ and rearranges to give three term quadratic</td><td>M1</td></tr>
<tr><td></td><td>Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$</td><td>Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$</td><td>M1</td></tr>
<tr><td></td><td>Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)</td><td>Obtains $y = 8, y = 1$ (may be on diagram)</td><td>A1</td></tr>
<tr><td></td><td>Substitutes their x into a given equation to give $y =$ (may be on diagram)</td><td>Substitutes their y into a given equation to give $x =$ (may be on diagram or in part (b))</td><td>M1</td></tr>
<tr><td></td><td>$y = 8, y = 1$</td><td>$x = 2, x = 9$</td><td>A1 (5)</td></tr>
<tr><td>(b)</td><td colspan="2">$\int (10x - x^2 - 8)\,dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{+c\}$</td><td>M1 A1 A1</td></tr>
<tr><td></td><td colspan="2">$\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x\right]_2^9 = (......) - (......)$</td><td>dM1</td></tr>
<tr><td></td><td colspan="2">$= 90 - \frac{4}{3} = 88\frac{2}{3}$ or $\frac{266}{3}$</td><td></td></tr>
<tr><td></td><td colspan="2">Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$</td><td>B1</td></tr>
<tr><td></td><td colspan="2">So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$</td><td>M1A1 cao
(7)</td></tr>
<tr><td></td><td colspan="2"></td><td>12 marks</td></tr>
</table>

Notes (a)	First **M1:** See scheme Second **M1**: See notes relating to solving quadratics Third **M1**: This may be awarded if one substitution is made Two correct Answers following tables of values, or from Graphical calculator are 5/5 **Just one pair of correct coordinates – no working or from table is M0M0A0M1A0**
(b)	**M1**: $x^n \to x^{n+1}$ for any one term. **1st A1**: at least two out of three terms correct **2nd A1**: All three correct **dM1**: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round (NB: If candidate **changes all signs** to get $\int(-10x+x^2+8)\,dx = -\frac{10x^2}{2}+\frac{x^3}{3}+8x\{+c\}$ This is M1 A1 A1 Then uses limits dM1 and trapezium is B1 Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3}-31.5$ is M0A0) **B1**: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2}\times8\times8-\frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6] **M1**: Their Area under curve – Their Area under line (if integrate both need same limits) **A1**: Accept 57.16recurring but not 57.16 **PTO for Alternative method**

Method 2 for (b)			
	Area of R $= \int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$	3rd M1 (in (b)): Uses difference between two functions in integral.	
	$\int_2^9 -x^2 + 11x - 18\,dx$	M: $x^n \to x^{n+1}$ for any one term.	M1
		A1 at least two out of these three simplified terms	A1
	$= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x\ \{+c\}$	Correct integration. (Ignore $+c$).	A1
	$\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (......) - (......)$	Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	This mark is implied by final answer which rounds to 57.2		B1
	See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:		M1
	$40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao		A1
			(7)

Special case of above method	$\int_2^9 x^2 - 11x + 18\,dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (......) - (......)$ This mark is implied by final answer which rounds to 57.2 (not -57.2) Difference of functions implied (see above expression) $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao	M1A1A1 DM1 B1 M1 A1 (7)

Special case of above method	$\int_2^9 x^2 - 11x + 18\,dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (......) - (......)$ This mark is implied by final answer which rounds to 57.2 (not -57.2) Difference of functions implied (see above expression) $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6}$ cao	M1A1A1 DM1 B1 M1 A1 (7)

Special Case 2	Integrates expression in y e.g. "$y^2 - 9y + 8 = 0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)	
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ only earns final M mark	

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28.

Question Number	Scheme		Marks
9.	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
(a)	$\{\text{Curve} = \text{Line}\} \Rightarrow -x^2 + 2x + 24 = x + 4$	Eliminating y correctly.	B1
	$x^2 - x - 20\ \{=0\} \Rightarrow (x-5)(x+4)\ \{=0\} \Rightarrow x = ...$	Attempt to solve a ***resulting*** quadratic to give x = their values.	M1
	So, $x = 5, -4$	Both $x = 5$ and $x = -4$.	A1
	So corresponding y-values are $y = 9$ and $y = 0$.	See notes below.	B1ft **[4]**
(b)	$\left\{\int(-x^2 + 2x + 24)\,\mathrm{d}x\right\} = \underline{-\frac{x^3}{3} + \frac{2x^2}{2} + 24x}\ \{+c$	M1: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms. 2nd A1 for <u>correct answer</u>.	M1A1A1
	$\left[-\frac{x^3}{3} + \frac{2x^2}{2} + 24x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + 25 + 120\right) - \left(\frac{64}{3} + 16 - 96\right) = \left(103\frac{1}{3}\right) - \left(-58\frac{2}{3}\right) = 162\right\}$		
	Area of $\Delta = \frac{1}{2}(9)(9) = 40.5$	Uses correct method for finding area of triangle.	M1
	So area of R is $162 - 40.5 = 121.5$	Area under curve – Area of triangle.	M1
		121.5	A1 oe **cao**
			[7]
			11

Question Number	Scheme	Marks
(a)	1st B1: For correctly eliminating either x or y. Candidates will usually write $-x^2 + 2x + 24 = x + 4$. This mark can be implied by the resulting quadratic. M1: For solving their quadratic (which must be different to $-x^2 + 2x + 24$) to give $x = ...$ See introduction for Method mark for solving a 3TQ. It must result from some attempt to eliminate one of the variables. A1: For both $x = 5$ and $x = -4$. 2nd B1ft: For correctly substituting their values of x in equation of line or parabola to give ***both correct ft*** y-values. (You may have to get your calculators out if they substitute their x into $y = -x^2 + 2x + 24$). **Note:** For $x = 5, -4 \Rightarrow y = 9$ and $y = 0 \Rightarrow$ eg. $(-4, 9)$ and $(5, 0)$, award B1 isw. If the candidate gives additional answers to $(-4, 0)$ and $(5, 9)$, then withhold the final B1 mark. **Special Case**: Award SC: B0M0A0B1 for $\{A\}(-4, 0)$. You may see this point marked on the diagram. **Note:** SC: B0M0A0B1 for solving $0 = -x^2 + 2x + 24$ to give $\{A\}(-4, 0)$ and/or $(6, 10)$. Note: Do not give marks for working in part (b) which would be creditable in part (a).	
(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $24 \to 24x$ is sufficient for M1. 1st A1 at least two out of three terms correctly integrated. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x = -4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Area of triangle = $\frac{1}{2}$(their x_2 − their x_1)(their y_2) or Area of triangle = $\int_{x_1}^{x_2} x + 4\ \{dx\}$. Where x_1 = their -4, x_2 = their 5 and y_2 = their y usually found in part (a). 4th M1: Area under curve – Area under triangle, where both Area under curve > 0 and Area under triangle > 0 and Area under curve > Area under triangle. 3rd A1: 121.5 or $\frac{243}{2}$ oe **cao.**	

Question Number	Scheme		Marks
***Aliter* 9.(b) Way 2**	Curve: $y = -x^2 + 2x + 24$, Line: $y = x + 4$		
	Area of R $= \int_{-4}^{5} (-x^2 + 2x + 24) - (x + 4)\,dx$	3rd M1: Uses integral of $(x + 4)$ with correct ft limits. 4th M1: Uses "curve" – "line" function with correct ft limits.	
	$= -\frac{x^3}{3} + \frac{x^2}{2} + 20x\ \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of three terms Correct answer (Ignore $+c$).	M1 A1ft A1
	$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5} = (......) - (......)$	Substitutes 5 and -4 (or ***their limits*** from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \left(\frac{64}{3} + 8 - 80\right) = \left(70\frac{5}{6}\right) - \left(-50\frac{2}{3}\right)\right\}$		
		See above working to decide to award 3rd M1 mark here: ***See above working to decide to award 4th M1 mark here:***	M1 M1
	So area of R is $= 121.5$	121.5	A1 oe **cao**
			[7] **11**

(b)	1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $20 \to 20x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft. Note this accuracy mark is ft in Way 2. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $-\frac{x^3}{3}+\frac{2x^2}{2}+24x-\left(\frac{x^2}{2}+4x\right)$. Note that $\frac{2x^2}{2}-\frac{x^2}{2}$ or $24x-4x$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -4 (and not 4 if the candidate has stated $x=-4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip! 3rd M1: Uses the integral of $(x+4)$ with correct ft limits of their x_1 and their x_2 (usually found in part (a)) $\{\text{where}\,(x_1, y_1)=(-4, 0)$ and $(x_2, y_2)=(5, 9).\}$ This mark is usually found in the first line of the candidate's working in part (b). 4th M1: Uses "curve" – "line" function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate's working in part (b). Allow $\int_{-4}^{5}(-x^2+2x+24)-x+4\,\{dx\}$ for this method mark. 3rd A1: 121.5 oe **cao.** **<u>Note: SPECIAL CASE for this alternative method</u>** Area of R $=\int_{-4}^{5}(x^2-x-20)\,dx=\left[\frac{x^3}{3}-\frac{x^2}{2}-20x\right]_{-4}^{5}=\left(\frac{125}{3}-\frac{25}{2}-100\right)-\left(-\frac{64}{3}-8+80\right)$ The working so far would score SPEICAL CASE M1A1A1M1M1M0A0. The candidate may then go on to state that $=\left(-70\frac{5}{6}\right)-\left(50\frac{2}{3}\right)=-\frac{243}{2}$ If the candidate then multiplies their answer by -1 then they would gain the 4th M1 and 121.5 would gain the final A1 mark.

Question Number	Scheme		Marks
Aliter	Curve: $y=-x^2+2x+24$, Line: $y=x+4$		
9. (a)	$\{\text{Curve}=\text{Line}\}\Rightarrow y=-(y-4)^2+2(y-4)+24$	Eliminating x correctly.	B1
Way 2	$y^2-9y\,\{=0\}\Rightarrow y(y-9)\,\{=0\}\Rightarrow y=.....$	Attempt to solve a resulting quadratic to give $y=$ their values.	M1
	So, $y=0, 9$	Both $y=0$ and $y=9$.	A1
	So corresponding y-values are $x=-4$ and $x=5$.	See notes below.	B1ft
			[4]
	2nd B1ft: For correctly substituting their values of y in equation of line or parabola to give ***both correct ft*** x-values.		
9. (b)	**<u>Alternative Methods for obtaining the M1 mark for use of limits:</u>** There are two alternative methods can candidates can apply for finding "162". **<u>Alternative 1:</u>** $\int_{-4}^{0}(-x^2+2x+24)\,dx+\int_{0}^{5}(-x^2+2x+24)\,dx$ $=\left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{-4}^{0}+\left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{0}^{5}$ $=(0)-\left(\frac{64}{3}+16-96\right)+\left(-\frac{125}{3}+25+120\right)-(0)$ $=\left(103\frac{1}{3}\right)-\left(-58\frac{2}{3}\right)=162$		

	Alternative 2:
	$$\int_{-4}^{6}(-x^2+2x+24)\,dx \;-\; \int_{5}^{6}(-x^2+2x+24)\,dx$$ $$=\left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{-4}^{6} - \left[-\frac{x^3}{3}+\frac{2x^2}{2}+24x\right]_{5}^{6}$$ $$=\left\{\left(-\frac{216}{3}+36+144\right)-\left(\frac{64}{3}+16-96\right)\right\} - \left\{\left(-\frac{216}{3}+36+144\right)-\left(-\frac{125}{3}+25+120\right)\right\}$$ $$=\left\{(108)-\left(-58\frac{2}{3}\right)\right\} - \left\{(108)-\left(103\frac{1}{3}\right)\right\}$$ $$=\left(166\frac{2}{3}\right)-\left(4\frac{2}{3}\right)=162$$

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29.

Question Number	Scheme		Marks
4. **(a)**	Seeing -1 and 5. (See note below.)		B1 (1)
(b)	$(x+1)(x-5)=\underline{x^2-4x-5}$ or $\underline{x^2-5x+x-5}$		B1
	$\int(x^2-4x-5)\,dx=\frac{x^3}{3}-\frac{4x^2}{2}-5x\;\{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft.	M1A1ft A1
	$\left[\frac{x^3}{3}-\frac{4x^2}{2}-5x\right]_{-1}^{5}=(......)-(......)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{\left(\frac{125}{3}-\frac{100}{2}-25\right)-\left(-\frac{1}{3}-2+5\right)=\left(-\frac{100}{3}\right)-\left(\frac{8}{3}\right)=-36\right\}$		
	Hence, Area $=36$	Final answer must be 36, not -36	A1 (6) **[7]**
	Notes		
(a)	B1: for -1 and 5. Note that $(-1,0)$ and $(5,0)$ are acceptable for B1. Also allow $(0,-1)$ and $(0,5)$ generously for B1. Note that if a candidate writes down that $A:(5,0)$, $B:(-1,0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x-axis of the graph.		

(b)	B1 for $x^2 - 4x - 5$ or $x^2 - 5x + x - 5$. If you believe that the candidate is applying the Way 2 method then $-x^2 + 4x + 5$ or $-x^2 + 5x - x + 5$ would then be fine for B1. 1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $-5 \to 5x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft from their multiplied out brackets. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x = -1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. 3rd A1: For a final answer of 36 , not -36. **Note:** An alternative method exists where the candidate states from the outset that Area $(R) = -\int_{-1}^{5}(x^2 - 4x + 5)\,dx$ is detailed in the Appendix.

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30.

Question Number	Scheme	Marks
8	(a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)	M1 A1
	At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*)	A1 cso
	N.B. The '= 0' must be seen at some stage to score the final mark.	
	Alternatively: (using $k = 28$) $\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1) 'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point.	(3)

(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$	M1 A1	
$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = ...$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$ (With limits 0 to 2, substitute the limit 2 into a 'changed function')	M1	
y-coordinate of $P = 8 - 40 + 56 = 24$ Allow if seen in part (a) (The B1 for 24 may be scored by implication from later working) Area of rectangle $= 2 \times$ (their y-coordinate of P)	B1	
Area of R = (their 48) $-\left(\text{their } \frac{100}{3}\right) = \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$ If the subtraction is the 'wrong way round', the final A mark is lost.	M1 A1	(6) 9
(a) M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen in part (a). (b) 1st M: $x^n \to cx^{n+1}$ (c constant, $c \neq 0$) for one term. Integrating the gradient function loses this M mark. 2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated). Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle. A1: Must be exact, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$. Alternative: (effectively finding area of rectangle by integration) $\int \{24 - (x^3 - 10x^2 + 28x)\}\,dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}\right)$, etc. This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.		

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31.

Question Number	Scheme	Marks
Q7 (a)	**Puts** $y = 0$ and attempts to solve quadratic e.g. $(x-4)(x-1)=0$ Points are (1,0) and (4, 0)	M1 A1 (2)
(b)	$x = 5$ gives $y = 25 - 25 + 4$ and so (5, 4) lies on the curve	B1cso (1)
(c)	$\int (x^2 - 5x + 4)\,dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x \quad (+c)$	M1A1 (2)
(d)	Area of triangle = $\frac{1}{2} \times 4 \times 4 = 8$ or $\int (x-1)\,dx = \frac{1}{2}x^2 - x$ with limits 1 and 5 to give 8	B1
	Area under the curve = $\int_4^5$ $\quad \frac{1}{3} \times 5^3 - \frac{5}{2} \times 5^2 + 4 \times 5 \quad \left[= -\frac{5}{6}\right]$	M1
	$\frac{1}{3} \times 4^3 - \frac{5}{2} \times 4^2 + 4 \times 4 \quad \left[= -\frac{8}{3}\right]$	M1
	$\int_4^5 = -\frac{5}{6} - -\frac{8}{3} = \frac{11}{6}$ or equivalent (allow 1.83 or 1.8 here)	A1 cao
	Area of R = $8 - \frac{11}{6} = 6\frac{1}{6}$ or $\frac{37}{6}$ or $6.1\dot{6}$ (not 6.17)	A1 cao (5)
		[10]

(a)	M1 for attempt to find L and M A1 Accept $x = 1$ and $x = 4$, then isw or accept L =(1,0) , M = (4,0) Do not accept L = 1, M = 4 nor (0 , 1), (0 , 4) (unless subsequent work) Do not need to distinguish L and M. Answers imply M1A1.	
(b)	See substitution, working should be shown, need conclusion which could be just $y = 4$ or a tick. Allow $y = 25 - 25 + 4 = 4$ But not 25 - 25 + 4 = 4. ($y = 4$ may appear at start) Usually 0 = 0 or 4 = 4 is B0	
(c)	M1 for attempt to integrate $x^2 \to kx^3, x \to kx^2$ or $4 \to 4x$ A1 for correct integration of all three terms (do not need constant) isw. Mark correct work when seen. So e.g. $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x$ is A1 then $2x^3 - 15x^2 + 24x$ would be ignored as subsequent work.	
(d)	B1 for this triangle only (not triangle LMN) 1st M1 for substituting 5 into their changed function 2nd M1 for substituting 4 into their changed function	
(d)	Alternative method: $\int_1^5 (x-1) - (x^2 - 5x + 4)\,dx + \int_1^4 x^2 - 5x + 4\,dx$ can lead to correct answer Constructs $\int_1^5 (x-1) - (x^2 - 5x + 4)\,dx$ is B1 M1 for substituting 5 and 1 and subtracting in first integral M1 for substituting 4 and 1 and subtracting in second integral A1 for answer to first integral i.e. $\frac{32}{3}$ (allow 10.7) and A1 for final answer as before..	

(d)	Another alternative $\int_4^5 (x-1)-(x^2-5x+4)\mathrm{d}x +$ *area of triangle LMP* Constructs $\int_4^5 (x-1)-(x^2-5x+4)\mathrm{d}x$ is B1 M1 for substituting 5 and 4 and subtracting in first integral M1 for complete method to find area of triangle (4.5) A1 for answer to first integral i.e. $\frac{5}{3}$ and A1 for final answer as before.
(d)	Could also use $\int_4^5 (4x-16)-(x^2-5x+4)\mathrm{d}x +$ *area of triangle LMN* Similar scheme to previous one. Triangle has area 6 A1 for finding Integral has value $\frac{1}{6}$ and A1 for final answer as before.