Quadratics- Marking Scheme

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
1	Attempts $f(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3)=0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3)=0$ method

$$\begin{array}{l} \quad\quad 3x^2+(2a-9)x+23-6a \\ x+3\,\overline{)\,3x^3+2ax^2-4x+5a} \\ \quad\quad \underline{3x^3+9x^2} \\ \quad\quad\quad (2a-9)x^2\ -4x \\ \quad\quad\quad \underline{(2a-9)x^2\ +(6a-27)x} \\ \quad\quad\quad\quad\quad\quad (23-6a)x+5a \\ \quad\quad\quad\quad\quad\quad (23-6a)x+69-18a \end{array}$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$6a-27+\frac{5a}{3}=-4$$

M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $f(\pm3)=0$. Allow for a linear equation in a leading to $a=...$. Don't be too concerned with the mechanics of this.

Via division accept $\begin{array}{l} \quad\quad 3x^2... \\ x+3\,\overline{)\,3x^3+2ax^2-4x+5a} \end{array}$ followed by a remainder in a set $=0\Rightarrow a=...$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a+12-81$ oe

A1: $a=3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

2.

Question	Scheme	Marks	AOs
5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $a=2$ & $b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1	2.2a
	(ii) $\text{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\text{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2\pm....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2\pm...$or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c\equiv ax^2+2abx+ab^2+c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x)=f(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x) \to g(x)$ is $2x^2+4x+9 \to 2(x-2)^2+4(x-2)+5$ So $g(x)=f(x-2)-4$
- $g(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7) \to (1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations. Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their }"7"}$
- Attempts to differentiate, sets $4x+4=0 \to x=-1$ and substitutes into $h(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=h(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<h(x)\leqslant 3$ Allow for $0<h\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<h(x)\leqslant \dfrac{21}{c}$

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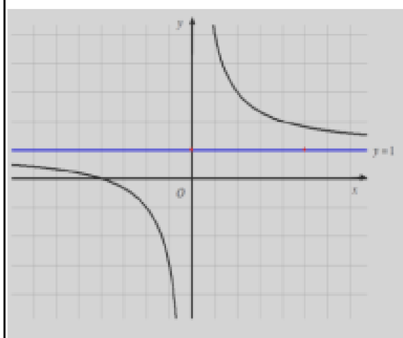
3.

Question	Scheme	Marks	AOs
7 (a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x)\Rightarrow k^2+1x=-2x^2+5x \Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

(a)

M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)

A1: Correct shape and position for both branches.
It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour

B1: Asymptote given as $y=1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.

(b)

M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x

A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips.
Condone if the order of the terms are different $2x^2+k^2-4x=0$

(c)

M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.**
If a, b and c are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$
Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0 \Rightarrow (x-1)^2=1-\frac{1}{2}k^2 \Rightarrow "1-\frac{1}{2}k^2"=0$

A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$
If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct

A1: $k=\pm\sqrt{2}$ and following correct a, b and c if stated

A solution via differentiation would be awarded as follows
M1: Sets the gradient of the curve $=-2 \Rightarrow -\frac{k^2}{x^2}=-2 \Rightarrow x=(\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2-4x+k^2=0$
A1: $2k^2=(\pm)2\sqrt{2}k$ oe
A1: $k=\pm\sqrt{2}$

4.

Question	Scheme	Marks	AOs
9 (a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Note: Only withhold the mark for a lack of tonnes, once, the first time that it occurs.

(a)

B1: 117 tonnes or 117 t.

(b)

B1: 1200 tonnes or 1200 t.

(c)

M1: Attempts $T_5 - T_4 = \{1200-3\times(5-20)^2\}-\{1200-3\times(4-20)^2\}$ May be implied by $525-432$

Condone for this mark an attempt at $T_4 - T_3 = \{1200-3\times(4-20)^2\}-\{1200-3\times(3-20)^2\}$

A1: 93 tonnes or 93 t

(d)

For one mark

Shows an appreciation of the model

- States $n \leqslant 20$ or $n < 20$
- Condone for one mark $n \leqslant 40$ or $n < 40$ **with** "the mass of tin mined cannot be negative" oe
- Condone for one mark $n = 40$ **with** a statement that "the mass of tin mined becomes 0" oe
- after 20 years the (total) amount of tin mined starts to go down (n may not be mentioned and total may be missing)
- after 20 years the (total) mass reaches a maximum value. (Similar to above)
- States T_{max} is reached when $n = 20$

For two marks

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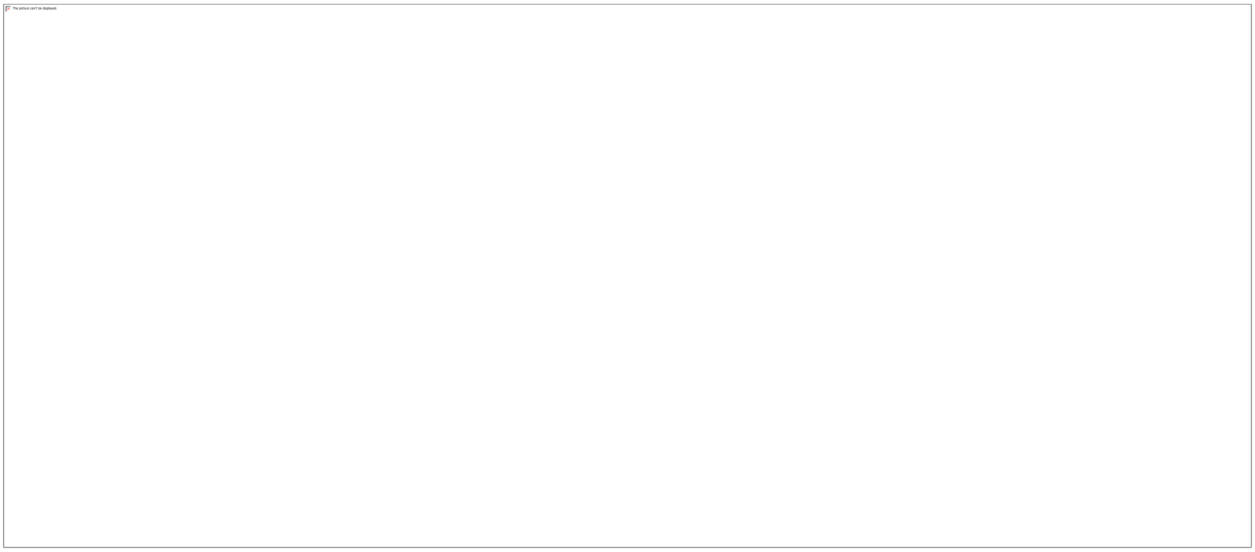

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May 2015 Mathematics Advanced Paper 1: Pure Mathematics 1

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May 2012 Mathematics Advanced Paper 1: Pure Mathematics 1

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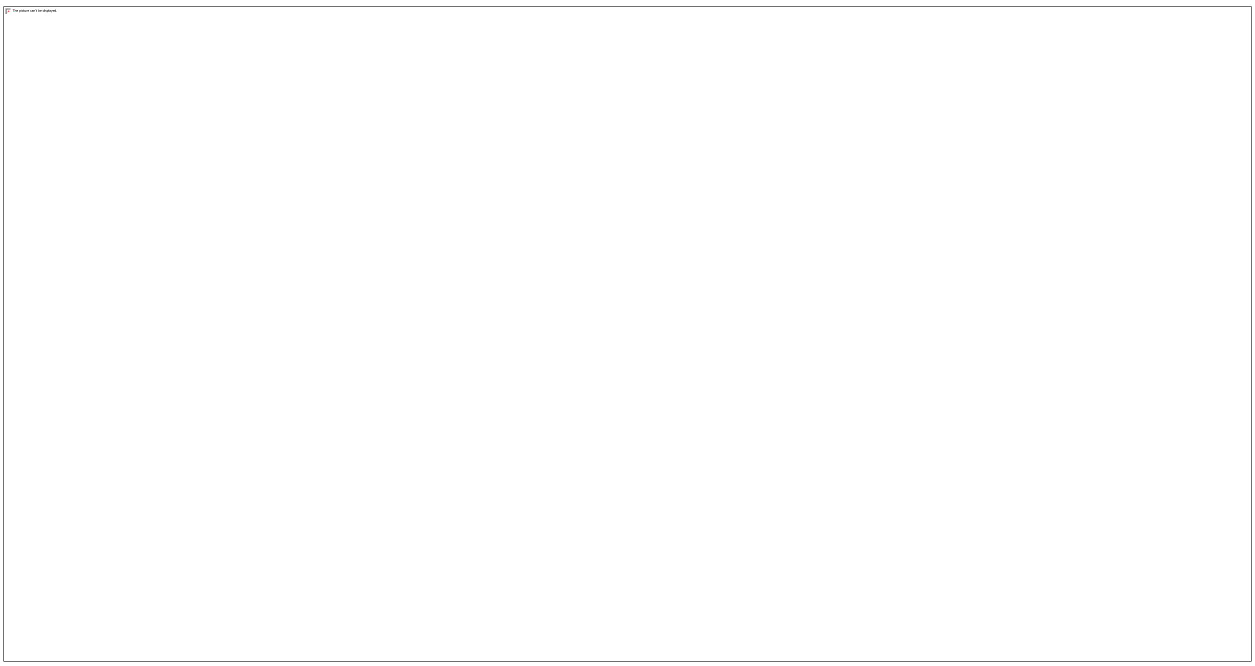

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May 2010 Mathematics Advanced Paper 1: Pure Mathematics 1

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Question Number	Scheme	Marks
Alt 1 **By Multiplication**	* $3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$	
	Compares the x^4 terms $a = 3$	B1
	Compares coefficients to obtain a numerical value of one **further** constant $-2 = b, \quad -5 = -4a + c \Rightarrow c = ..,$	M1
	Two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
	All four of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
		(4 marks)

Notes for Question 2

B1 Stating $a = 3$. This can also be scored for writing $3x^4 = ax^4$

M1 Multiply out expression given to get *. Condone slips only on signs of either expression.
Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either b or c
The method may be implied by a correct additional constant to a.

A1 Achieving two of $b = -2$ $c = 7$ $d = -8$ $e = 24$

A1 Achieving all of $b = -2$ $c = 7$ $d = -8$ and $e = 24$