Differentiation- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\dfrac{dy}{dx} < 0$

(1)

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

2.

Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

3.

A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

4.

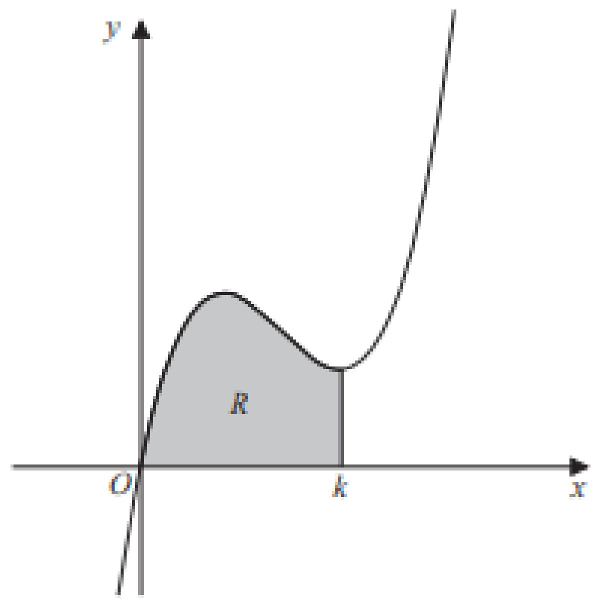

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\frac{256}{3}$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(7)

May 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

5.

A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey £C when the lorry is driven at a steady speed of v kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60$$

(a) Find, according to this model,

(i) the value of v that minimises the cost of the journey,

(ii) the minimum cost of the journey.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(b) Prove by using $\frac{d^2C}{dv^2}$ that the cost is minimised at the speed found in (a)(i).

(2)

(c) State one limitation of this model.

(1)

6.

Prove, from first principles, that the derivative of x^3 is $3x^2$

(4)

May 2017 Mathematics Advanced Paper 1: Pure Mathematics 1

7.

Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \quad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where a is a rational number.

(5)

8.

The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point P (4, −8) lies on C,

(*a*) find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(*b*) Find f(x), giving each term in its simplest form.

(5)

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 1

9.

Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0,$$

find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

10.

The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(*a*) Find $\frac{dy}{dx}$.

(2)

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$.

Find

(*b*) the value of k,

(4)

(*c*) the value of the y coordinate of P,

(2)

(*d*) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

May 2015 Mathematics Advanced Paper 1: Pure Mathematics 1

11.

Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\int y \, dx$.

(3)

12.

The curve C has equation

$$y = \frac{(x^2+4)(x-3)}{2x}, \quad x \neq 0.$$

(*a*) Find $\frac{dy}{dx}$ in its simplest form.

(5)

(*b*) Find an equation of the tangent to C at the point where $x = -1$.

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

13.

A curve with equation $y = f(x)$ passes through the point $(4, 9)$.

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0,$$

(*a*) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$.

(*b*) Find the x-coordinate of P.

(5)

May 2014 Mathematics Advanced Paper 1: Pure Mathematics 1

14.

Differentiate with respect to x, giving each answer in its simplest form,

(*a*) $(1 - 2x)^2$,

(3)

(*b*) $\dfrac{x^5 + 6\sqrt{x}}{2x^2}$.

(4)

15.

Given that $f(x) = 2x^2 + 8x + 3$,

(*a*) find the value of the discriminant of $f(x)$.

(2)

(*b*) Express $f(x)$ in the form $p(x + q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(*c*) Calculate the value of c.

(5)

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 1

16.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0.$$

(*a*) Show that $f'(x) = 9x^{-2} + A + Bx^2$, where A and B are constants to be found.

(3)

(*b*) Find $f''(x)$.

(2)

Given that the point $(-3, 10)$ lies on the curve with equation $y = f(x)$,

(*c*) find $f(x)$.

(5)

17.

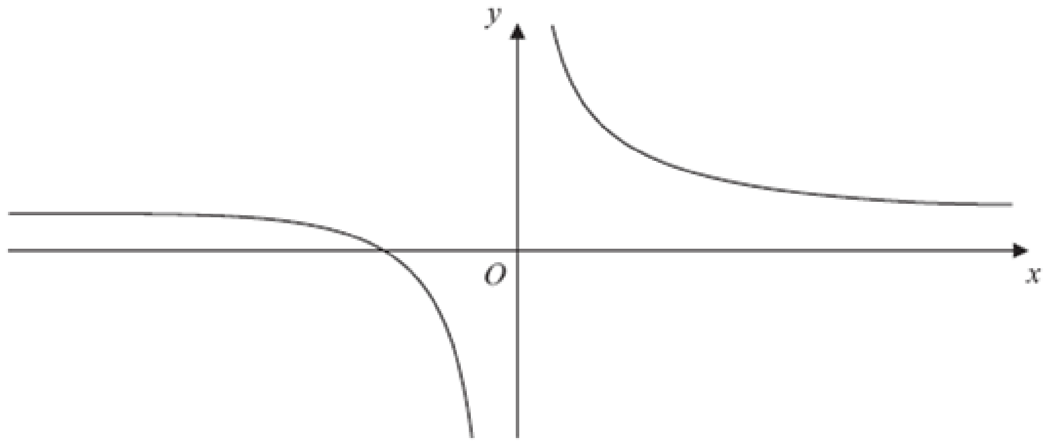

Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x-axis.
(1)

(b) Give the equations of the asymptotes to H.
(2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$.
(5)

This normal crosses the x-axis at A and the y-axis at B.

(d) Find the length of the line segment AB. Give your answer as a surd.
(3)

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 1

18.

11. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0.$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.
(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$.

(*b*) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$.

(*c*) Find the coordinates of Q.

(5)

May 2012 Mathematics Advanced Paper 1: Pure Mathematics 1

19.

4.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3.$$

(*a*) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(4)

(*b*) Find $\frac{d^2y}{dx^2}$.

(2)

20.

7. The point P (4, −1) lies on the curve C with equation $y = f(x)$, $x > 0$, and

$$f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3.$$

(*a*) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

(*b*) Find f(x).

(4)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 1

21.

1. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\int y \; dx$.

(3)

22.

8. The curve C_1 has equation

$$y = x^2(x + 2).$$

(*a*) Find $\frac{dy}{dx}$.

(2)

(*b*) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis.

(3)

(*c*) Find the gradient of C_1 at each point where C_1 meets the x-axis.

(2)

The curve C_2 has equation

$$y = (x - k)^2(x - k + 2),$$

where k is a constant and $k > 2$.

(*d*) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes.

(3)

23.

10.

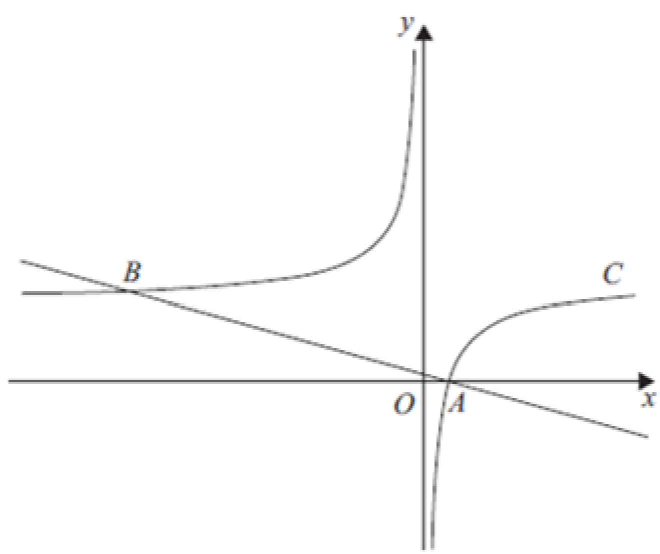

Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y = 2 - \frac{1}{x}, \qquad x \neq 0.$$

The curve crosses the x-axis at the point A.

(*a*) Find the coordinates of A.

(1)

(*b*) Show that the equation of the normal to C at A can be written as

$$2x + 8y - 1 = 0.$$

(6)

The normal to C at A meets C again at the point B, as shown in Figure 2.

(*c*) Find the coordinates of B.

(4)

May 2011 Mathematics Advanced Paper 1: Pure Mathematics 1

24.

2. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(*a*) $\frac{dy}{dx}$,

(3)

(*b*) $\int y \; dx$.

(4)

25.

10. The curve C has equation

$$y = (x + 1)(x + 3)^2.$$

(*a*) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(*b*) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(*c*) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(*d*) Find the x-coordinate of B.

(3)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 1

26.

11. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(*a*) Find $\frac{dy}{dx}$.

(4)

(*b*) Show that the point $P(4, -8)$ lies on C.

(2)

(*c*) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

May 2010 Mathematics Advanced Paper 1: Pure Mathematics 1

27.

7. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \qquad x > 0,$$

find $\frac{dy}{dx}$.

(6)

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 1

28.

1. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

(3)

29.

6. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(*a*) Find $\frac{dy}{dx}$ in its simplest form.

(4)

(*b*) Find an equation of the tangent to C at the point where $x = 2$.

(4)

May 2014 Mathematics Advanced Paper 1: Pure Mathematics 2

30.

10.

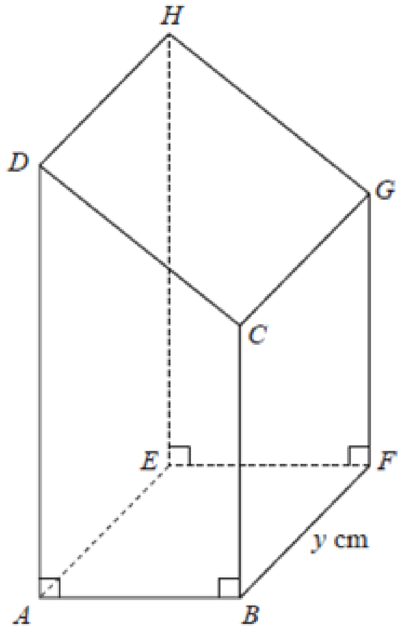

Figure 4

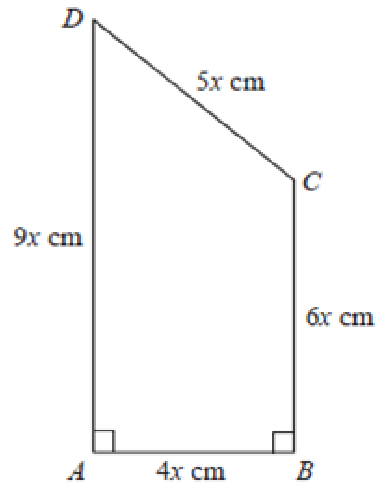

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.

The angle $DAB = 90°$ and the angle $ABC = 90°$. The volume of the letter box is 9600 cm^3.

(*a*) Show that $y = \dfrac{320}{x^2}$.

(2)

(*b*) Hence show that the surface area of the letter box, S cm^2, is given by $S = 60x^2 + \dfrac{7680}{x}$.

(4)

(*c*) Use calculus to find the minimum value of S.

(6)

(*d*) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

31.

9. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20, \qquad x > 0,$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

32.

8. The curve C has equation $y = 6 - 3x - \dfrac{4}{x^3}$, $x \neq 0$.

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$.

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\dfrac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

May 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

33.

8.

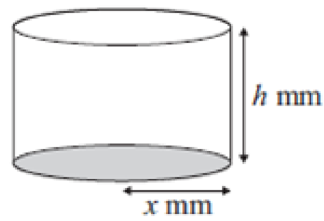

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(*a*) express h in terms of x,

(1)

(*b*) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$.

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(*c*) Use calculus to find the value of x for which A is a minimum.

(5)

(*d*) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(*e*) Show that this value of A is a minimum.

(2)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

34.

10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

(*a*) Find $\dfrac{dV}{dx}$.

(4)

(*b*) Hence find the maximum volume of the box.

(4)

(*c*) Use calculus to justify that the volume that you found in part (*b*) is a maximum.

(2)

Jun 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

35.

3. $y = x^2 - k\sqrt{x}$, where k is a constant.

(*a*) Find $\frac{dy}{dx}$.

(2)

(*b*) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 2

36.

9. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10,\ x > 0$.

(*a*) Use calculus to find the coordinates of the turning point on C.

(7)

(*b*) Find $\frac{d^2y}{dx^2}$.

(2)

(*c*) State the nature of the turning point.

(1)