

Measures of location and spread - Answers

May 2014 Mathematics Advanced Paper 1: Statistics 1

1.

Question Number	Scheme	Marks
2	$\text{mean} = \frac{60.8 + 20}{1.4} \quad \text{or} \quad 60.8 = 1.4x - 20 \quad (\text{o.e.})$ $= 57.7142\dots \quad \text{awrt } 57.7$	M1 A1
	$\text{standard deviation} = \frac{6.60}{1.4} \quad \text{or} \quad 6.60 = 1.4x$ $= 4.7142\dots \quad \text{awrt } 4.71$	M1 A1 (4) Total 4
Notes		
<p>1st M1 sub. 60.8 for y into a correct equation. Allow use of x or any other letter or expression for mean</p> <p>1st A1 for awrt 57.7 or $\frac{404}{7}$ (o.e.). Correct answer only is 2/2</p> <p>2nd M1 sub. 6.60 or 6.6 for y and ignoring the 20 Allow use of x or any other letter or expression for st. dev. $6.60^2 = 1.4^2 x^2$ is M0 until we see them take a square root.</p> <p>2nd A1 for awrt 4.71 or $\frac{33}{7}$ (o.e.). Correct answer only is 2/2</p>		

2.

Question Number	Scheme	Marks
<p>4 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>60</p> <p>$Q_1 = 46$ $Q_2 = 56$ $Q_3 = 64$</p> <p>mean = 55.48.... or $\frac{2497}{45}$ awrt 55.5</p> <p>$sd = \sqrt{\frac{143369}{45} - \left(\frac{2497}{45}\right)^2}$ = 10.342... ($s = 10.459..$) anything which rounds to 10.3 (or $s = 10.5$)</p> <p>Mean < median < mode or $Q_2 - Q_1 > Q_3 - Q_2$ with or without their numbers or median closer to upper quartile (than lower quartile) or (mean-median)/sd < 0; negative skew;</p> <p>mean = $(55 - 5) \times 0.9$ = 45 sd = 10×0.9 = 9</p>	<p>B1 (1)</p> <p>B1 B1 B1 (3)</p> <p>B1 M1 A1 (3)</p> <p>B1 B1dep (2)</p> <p>M1 A1 M1 A1 (4)</p> <p>Total 13</p>
<p>NOTES</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>B1 60 only</p> <p>Award each B1 for correct answer only in this order.</p> <p>M1 for use of correct formula, including square root. Correct answers with no working B1M1A1.</p> <p>B1 any correct comparison of a pair of mean, median and mode using their values. B1 for 'negative skew' or allow (almost) symmetrical dependent upon correct reason.</p> <p>M1 for $(55 \text{ or } 55.5 - 5) \times 0.9$ A1 for the correct answer only. M1 for $(10 \text{ or } 10.3 \text{ or } 10.5) \times 0.9$ A1 for the correct answer only.</p>	

3.

Question Number	Scheme	Marks
<p>2.</p> <p>(a)</p>	$2.8 + 5.6 + 2.3 + 9.4 + 0.5 + 1.8 + 84.6 = 107$ $\text{mean} = 107 / 28 (= 3.821\dots)$ <p style="text-align: right;">(awrt 3.8)</p>	<p>M1</p> <p>A1</p> <p style="text-align: right;">(2)</p>
<p>(b)</p>	<p>It will have no effect since one is 4.5 under what it should be and the other is 4.5 above what it should be.</p>	<p>B1</p> <p>dB1</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;">[4]</p>
Notes		
<p>(a)</p>	<p>M1 for a clear attempt to add the two sums. Accept a full expression or $2.8 + 5.6 + \dots + 84.6 = x$ where $100 < x < 110$ i.e. seeing at least two correct terms of Keith's and the 84.6 with a slip.</p> <p>A1 for awrt 3.8 (Condone 1 dp/2sf here since data is given to 1 dp or 2 sf)</p> <p>Accept $\frac{107}{28}$ or $3\frac{23}{28}$ or any exact equivalent</p> <p>Correct answer implies M1A1</p>	
<p>(b)</p>	<p>1st B1 for clearly stating that it will have no effect. ("roughly the same" is B0 B0)</p> <p>2nd dB1 for a supporting reason that mentions the fact that the increase and decrease are the same and gives some numerical value(s) to support this.</p> <p>e.g. Sum of Keith's observations is still 22.4 (or mean is still 3.2)</p> <p><u>or</u> Sum is still 107</p> <p><u>or</u> $9.4 - 4.9 = 5 - 0.5$ (o.e.)</p> <p>This second B1 is dependent on their saying there is no effect so B0B1 is not possible.</p>	

4.

Question Number	Scheme	Marks
5.		
(a)	Median = $32/2 = 16^{\text{th}}$ term (16.5) $\frac{x-39.5}{49.5-39.5} = \frac{16-14}{25-14}$ or $x = 39.5 + \left(\frac{2}{11} \times 10\right)$ Median = 41.3 (use of $n + 1$ gives 41.8) (awrt 41.3)	M1 A1 (2)
(b)	Mean = $\frac{1414}{32} = 44.1875$ (awrt 44.2) Standard deviation = $\sqrt{\frac{69378}{32} - \left(\frac{1414}{32}\right)^2}$ = 14.7 (or $s = 14.9$)	B1 M1 A1 (3)
(c)	mean > median therefore <u>positive skew</u>	B1ft B1ft (2) [7]
Notes		
(a)	M1 for an attempt to use interpolation to find the median. Condone use of 39 or 40 for 39.5 e.g. allow $39 + \frac{2}{11} \times 10$ (o.e.) or $40 + \frac{2}{11} \times 10$ (o.e.) to score M1A0 but must have the 10 A1 for awrt 41.3 (or awrt 41.8 if using $(n + 1)$)	
(b)	B1 for awrt 44.2 M1 for a correct expression including square root. (Allow ft of their mean) A1 for awrt 14.7 (If using s for awrt 14.9) You may see $\sum t = 1339 \rightarrow \bar{t} = 41.8$ and $\sum t^2 = 62928 \rightarrow \sigma 14.7$ or $s = 14.9$ Mid-points this scores B0 for the mean but can score M1 for a correct st.dev expression and A1 for ans. Correct answer only in (a) and (b) can score full marks but check $(n + 1)$ case in (a)	
(c)	Quartiles 1 st B1ft for a correct comparison of their mean and their median (may be in a formula) Calculating median – mean as negative is OK for this B1 but must say +ve skew for 2 nd B1 Only allow comparison to be ≈ 0 if $ \text{mean} - \text{median} \leq 0.5$ 2 nd B1ft for a correct description of skewness <u>based on their values of mean and median.</u> ft their values for mean and median not their previous calculation/comparison Must be compatible with their previous comparison (if they have one) “Positive skew” with no reason is B0B1 provided you can see their values that imply that. Description should be “positive” or “negative” or “no” skew or “symmetric” “Positive correlation” is B0 1 st B1ft if $Q_1 = \text{awrt } 32$ <u>and</u> $Q_3 = \text{awrt } 49$ seen and a correct comparison made. ft Q_2 2 nd B1ft if $Q_1 = \text{awrt } 32$ <u>or</u> $Q_3 = \text{awrt } 49$ seen and a correct description based on their quartiles and their comparison is made. (Should get “negative skew”)	

5.

Question Number	Scheme	Marks
Q5 (a)	23, 35.5 (may be in the table)	B1 B1 (2)
(b)	Width of 10 units is 4 cm so width of 5 units is 2 cm	B1
	Height = $2.6 \times 4 = \mathbf{10.4 \text{ cm}}$	M1 A1 (3)
(c)	$\sum fx = 1316.5 \Rightarrow \bar{x} = \frac{1316.5}{56} =$ awrt 23.5	M1 A1
	$\sum fx^2 = 37378.25$ can be implied	B1
	So $\sigma = \sqrt{\frac{37378.25}{56} - \bar{x}^2} =$ awrt 10.7 allow $s = 10.8$	M1 A1 (5)
(d)	$Q_2 = (20.5) + \frac{(28-21)}{11} \times 5 = 23.68\dots$ awrt 23.7 or 23.9	M1 A1 (2)
(e)	$Q_3 - Q_2 = 5.6, Q_2 - Q_1 = 7.9$ (or $\bar{x} < Q_2$) [7.9 > 5.6 so] negative skew	M1 A1 (2)
Total 14		
(b)	M1 for their width x their height=20.8. Without labels assume width first, height second and award marks accordingly.	
(c)	1 st M1 for reasonable attempt at $\sum x$ and /56 2 nd M1 for a method for σ or s , $\sqrt{\quad}$ is required Typical errors $\sum (fx)^2 = 354806.3$ M0, $\sum f^2x = 13922.5$ M0 and $(\sum fx)^2 = 1733172$ M0 Correct answers only, award full marks.	
(d)	Use of $\sum f(x - \bar{x})^2 =$ awrt 6428.75 for B1 lcb can be 20, 20.5 or 21, width can be 4 or 5 and the fraction part of the formula correct for M1 - Allow 28.5 in fraction that gives awrt 23.9 for M1A1	
(e)	M1 for attempting a test for skewness using quartiles or mean and median. Provided median greater than 22.55 and less than 29.3 award for M1 for $Q_3 - Q_2 < Q_2 - Q_1$ without values as a valid reason. SC Accept mean close to median and no skew oe for M1A1	

6.

Question Number	Scheme	Marks
Q3 (a)	2.75 or $2\frac{3}{4}$, 5.5 or 5.50 or $5\frac{1}{2}$	B1 B1 (2)
Q3 (b)	Mean birth weight = $\frac{4841}{1500} = 3.227\bar{3}$	awrt 3.23 M1 A1 (2)
Q3 (c)	Standard deviation = $\sqrt{\frac{15889.5}{1500} - \left(\frac{4841}{1500}\right)^2} = 0.421093\dots$ or $s = 0.4212337\dots$	M1 A1ft A1 (3)
Q3 (d)	$Q_2 = 3.00 + \frac{403}{820} \times 0.5 = 3.2457\dots$ (allow 403.5..... \rightarrow 3.25)	M1 A1 (2)
Q3 (e)	Mean(3.23) < Median(3.25) (or very close) Negative Skew (or symmetrical)	B1ft dB1ft (2) Total [11]
Q3 (b)	M1 for a correct expression for mean. Answer only scores both.	
Q3 (c)	M1 for a correct expression (ft their mean) for sd or variance. Condone mis-labelling eg sd=... with no square root or no labelling 1 st A1ft for a correct expression (ft their mean) including square root and no mis-labelling Allow 1 st A1 for $\sigma^2 = 0.177\dots \rightarrow \sigma = 0.42\dots$ 2 nd A1 for awrt 0.421. Answer only scores 3/3	
Q3 (d)	M1 for a correct expression (allow 403.5 i.e. use of $n + 1$) but must have 3.00, 820 and 0.5 A1 for awrt 3.25 provided M1 is scored. NB 3.25 with no working scores 0/2 as some candidates think mode is 3.25.	
Q3 (e)	1 st B1ft for a comparison of their mean and median (may be in a formula but if $\pm(\text{mean} - \text{median})$ is calculated that's OK. We are not checking the <u>value</u> but the <u>sign</u> must be consistent.) Also allow for use of quartiles <u>provided correct values seen</u> : $Q_1 = 3.02, Q_3 = 3.47$ [They should get $(0.22 =) Q_3 - Q_2 < Q_2 - Q_1 (= 0.23)$ and say (slight) negative skew or symmetric] 2 nd dB1ft for a compatible comment based on their comparison. Dependent upon a suitable, correct comparison. Mention of "correlation" rather than "skewness" loses this mark.	