Algebraic Methods- Mark Scheme

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 4

1.

Question Number	Scheme		Marks
3.	**Method 1: Using one identity**		
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \frac{B}{(x+2)} + \frac{C}{(3x-1)}$		
	$A = 3$	their constant term = 3	B1
	$9x^2+20x-10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x^2: \; 9 = 3A, \quad x: \; 20 = 5A + 3B + C$ constant: $-10 = -2A - B + 2C$ or $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using a correct identity.	A1
			[4]
	Method 2: Long Division		
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{5x-4}{(x+2)(3x-1)}$	their constant term = 3	B1
	So, $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$		
	$5x - 4 \equiv B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either $x: \; 5 = 3B + C$, constant: $-4 = -B + 2C$ or $x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1

	$x = \frac{1}{3} \Rightarrow \frac{5}{3} - 4 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$ So, $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$	Correct values for their B and their C, which are found using $5x - 4 \equiv B(3x-1) + C(x+2)$	A1 **[4]** **4**
	NOTE: This question appears as B1M1A1A1 on ePEN, but is now marked as B1B1M1A1. **BE CAREFUL!:** Candidates will assign ***their own*** "A, B and C" for this question. **1st B1:** Their constant term must be equal to 3 for this mark. **2nd B1:** Forming a correct identity. This can be implied by later working. **M1:** Attempts to find the value of either one of their B or their C from their identity. This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously. **A1:** Correct values for their B and their C, which are found using a correct identity. **Note and beware:** A number of candidates who write $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv \frac{A}{(x+2)} + \frac{B}{(3x-1)}$, leading to $9x^2 + 20x - 10 \equiv A(3x-1) + B(x+2)$, leading to $A = 2$ and $B = -1$ will gain a maximum of B0B0M1A0 for attempting to find either their A or their B from $9x^2 + 20x - 10 \equiv A(3x-1) + B(x+2)$. **Note:** The correct partial fraction from no working scores B1B1M1A1. **Note:** The final A1 is effectively dependent upon the second B1.		

3. ctd	**Note:** You can imply the 2nd B1 from either $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv \frac{A(x+2)(3x-1) + B(3x-1) + C(x+2)}{(x+2)(3x-1)}$ or $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B(3x-1) + C(x+2)}{(x+2)(3x-1)}$	
	Alternative Method 1: Initially dividing by (x + 2) $\frac{9x^2 + 20x - 10}{"(x+2)"(3x-1)} \equiv \frac{9x+2}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$ $\equiv 3 + \frac{5}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$	**B1:** their constant term = 3
	So, $\frac{-14}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$ $-14 \equiv B(3x-1) + C(x+2)$ $\Rightarrow B = 2, C = -6$	**B1:** Forming a correct identity. **M1:** Attempts to find either one of their B or their C from their identity.
	So, $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv 3 + \frac{5}{(3x-1)} + \frac{2}{(x+2)} - \frac{6}{(3x-1)}$ and $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$	**A1:** Correct answer in partial fractions.

Alternative Method 2: Initially dividing by (3x - 1)

$$\frac{9x^2+20x-10}{(x+2)"(3x-1)"} \equiv \frac{3x+\frac{23}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$$

$$\equiv 3 + \frac{\frac{5}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$$

B1: their constant term $= 3$

So, $\frac{-\frac{7}{3}}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$

$-\frac{7}{3} \equiv B(3x-1) + C(x+2)$

B1: Forming a correct identity.

$\Rightarrow B = \frac{1}{3},\ C = -1$

M1: Attempts to find either one of their B or their C from their identity.

So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{\frac{5}{3}}{(x+2)} + \frac{\frac{1}{3}}{(x+2)} - \frac{1}{(3x-1)}$

and $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$

A1: Correct answer in partial fractions.

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2.

Question Number	Scheme		Marks	
1.	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$		B1	
	$x \to 1 \qquad 9 = 3B \Rightarrow B = 3$		M1	
	$x \to -\frac{1}{2} \qquad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of A, B, C	A1	
	x^2 terms $\qquad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1	**(4)** **[4]**
	Alternatives for finding A.			
	x terms $\qquad 0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \Rightarrow A = 4$			

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3.

Question Number	Scheme	Marks
5.	(a) $A = 2$	B1
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$	
	$x \to 1 \qquad -3 = 3B \Rightarrow B = -1$	M1 A1
	$x \to -2 \qquad -12 = -3C \Rightarrow C = 4$	A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$	M1
	$(1-x)^{-1} = 1 + x + x^2 + \ldots$	B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ldots$	B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ldots$	M1
	$= 5 + \ldots$ ft their $A - B + \frac{1}{2}C$	A1 ft
	$= \ldots + \dfrac{3}{2}x^2 + \ldots$ $0x$ stated or implied	A1 A1 **(7)**
		[11]