Binomial Expansion- Mark Scheme

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 2

1.

Question Number	Scheme		Marks
5.	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $\mathrm{f}(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a)	First term of 16 in their final series		B1
Way 1	At least one of $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a)	$(2-9x)^4 = (4 - 36x + 81x^2)(4 - 36x + 81x^2)$		
		First term of 16 in their final series	B1
Way 2	$= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$	Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x or at least 2 terms in x^2.	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 3**	$\{(2-9x)^4 =\}\ 2^4\left(1 - \frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1 + 4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2 + ...\right)$	At least one of $(4 \times ... \times x)$ or $\left(\frac{4(3)}{2} \times ... \times x^2\right)$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]

	Parts (b), (c) and (d) may be marked together		
(b)	$A = "16"$	Follow through their value from (a)	B1ft
			[1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + ...\})$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow k = \frac{7}{2}$	$k = \frac{7}{2}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right)$; $= 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

	Question 5 Notes	
(a) Ways 1 and 3	B1 cao	16
	M1	Correct binomial coefficient associated with correct power of x *i.e* $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$ They may have 4 and 6 or 4 and $\frac{4(3)}{2}$ or even $\binom{4}{1}$ and $\binom{4}{2}$ as their coefficients. Allow missing signs and brackets for the M marks.
	1st A1	At least one of $-288x$ or $+1944x^2$ (allow +- 288x)
	2nd A1	Both $-288x$ **and** $+1944x^2$ (May list terms separated by commas) Also full marks for correct answer with no working here. Again allow +- 288x
	Note	If the candidate then divides their final correct answer through by 8 or any other common factor then isw and mark correct series when first seen. So (a) B1M1A1A1 .It is likely that this approach will be followed by (b) B0, (c) M1A0, (d) M1A0 if they continue with their new series e.g. $2-36x+283x^2+...$ (Do not ft the value 2 as a mark was awarded for 16)
Way 2b	Special Case	**Slight Variation on the solution given in the scheme**

$$(2-9x)^4 = (2-9x)(2-9x)(4-36x+81x^2)$$
$$= (2-9x)(8-108x+486x^2+...)$$

	First term of 16	**B1**
$= 16 - 216x + 972x^2 - 72x + 972x^2$	Multiplies out to give either 2 terms in x or 2 terms in x^2.	**M1**
$= (16) - 288x + 1944x^2 + ...$	At least one of $-288x$ or $+1944x^2$	**A1**
	Both $-288x$ **and** $+1944x^2$	**A1**

		Parts (b), (c) and (d) may be marked together.
(b)	B1ft	Must **identify** $A = 16$ or $A =$ *their* constant term found in part (a). Or may write just 16 if this is clearly their answer to part (b). If they expand their series and have 16 as first term of a series it is not sufficient for this mark.
(c)	M1	Candidate shows intention to multiply (1+kx) by part of their series from (a) e.g. Just $(1+kx)(16-288x+...)$ or $(1+kx)(16-288x+1944x^2+...)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find **two terms** (or coefficients) in x. i.e. f.t. their $-288x+16kx$ N.B. $-288kx = -232x$ with no evidence of brackets is M0 – allow copying slips, or use of factored series, as this is a method mark
	A1	$k = \frac{7}{2}$ o.e. so 3.5 is acceptable
(d)	M1	Multiplies out their $(1+kx)(16-288x+1944x^2+...)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find B using **these two** terms and a numerical value of k.
	A1	936
	Note	Award A0 for $B = 936x^2$ But allow A1 for $B = 936x^2$ **followed by** $B = 936$ and treat this as a correction Correct answers in parts (c) and (d) with no method shown may be awarded full credit.

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2.

Question Number	Scheme		Marks
3. (a)	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\{(2-3x)^6\} = (2)^6 + \underline{{}^6C_1}(2)^5(-3\underline{x}) + \underline{{}^6C_2}(2)^4(-3\underline{x})^2 + ...$		<u>M1</u>
	M1: $({}^6C_1 \times ... \times x)$ or $({}^6C_2 \times ... \times x^2)$. For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. <u>Ignore any extra terms.</u>		
	${}^6C_1 2^5 - 3x + {}^6C_2 2^4 - 3x^2 + ...$ Scores M0 unless later work implies a correct method		
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1
			[4]
(a) Way 2	$(2-3x)^6 = 64 + ...$	64 seen as the only constant term in their expansion.	B1
	$\left(1-\frac{3}{2}x\right)^6 = 1 + \underline{{}^6C_1}\left(\frac{-3}{2}\underline{x}\right) + \underline{{}^6C_2}\left(\frac{-3}{2}\underline{x}\right)^2 + ...$	M1: $({}^6C_1 \times ... \times x)$ or $({}^6C_2 \times ... \times x^2)$. For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or -3) may be wrong or missing. The terms can be "listed" rather than added. Ignore any extra terms.	<u>M1</u>
	$= 64 - 576x + 2160x^2 + ...$	A1: Either $-576x$ or $2160x^2$ (Allow $+ -576x$ here) A1: Both $-576x$ and $2160x^2$ (Do not allow $+ -576x$ here)	A1A1

(b)	Candidate writes down $\left(1+\frac{x}{2}\right)\times$(their part (a) answer, at least up to the term in x). (Condone missing brackets) $\left(1+\frac{x}{2}\right)(64-576x+...)$ or $\left(1+\frac{x}{2}\right)(64-576x+2160x^2+...)$ or $\left(1+\frac{x}{2}\right)64-\left(1+\frac{x}{2}\right)576x$ or $\left(1+\frac{x}{2}\right)64-\left(1+\frac{x}{2}\right)576x+\left(1+\frac{x}{2}\right)2160x^2$ or $64+32x, -576x-288x^2,\ 2160x^2+1080x^3$ are fine.	M1
	$=64-544x+1872x^2+...$ — A1: At least 2 terms correct as shown. (Allow $+-544x$ here) A1: $64-544x+1872x^2$ The terms can be "listed" rather than added. Ignore any extra terms.	A1A1
		[3]
		Total 7
	SC: If a candidate expands in descending powers of *x*, only the M marks are available	
	e.g. $\{(2-3x)^6\}=(-3x)^6+{}^6C_1(2)^1(-3x)^5+{}^6C_2(2)^2(-3x)^4+...$	

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3.

Question Number	Scheme	Marks
2. (a)	$(2+3x)^4$ - Mark (a) and (b) together $2^4+{}^4C_1 2^3(3x)+{}^4C_2 2^2(3x)^2+{}^4C_3 2^1(3x)^3+(3x)^4$ First term of 16 $\left({}^4C_1\times...\times x\right)+\left({}^4C_2\times...\times x^2\right)+\left({}^4C_3\times...\times x^3\right)+\left({}^4C_4\times...\times x^4\right)$ $=(16+)\ 96x+216x^2+216x^3+81x^4$ Must use Binomial – otherwise A0, A0	 B1 M1 A1 A1 **(4)**
(b)	$(2-3x)^4=16-96x+216x^2-216x^3+81x^4$	B1ft **(1)** **5**
Alternative method (a)	$(2+3x)^4=2^4(1+\frac{3x}{2})^4$ $2^4(1+{}^4C_1(\frac{3x}{2})+{}^4C_2(\frac{3x}{2})^2+{}^4C_3(\frac{3x}{2})^3+(\frac{3x}{2})^4)$ Scheme is applied exactly as before	

	Notes for Question 2
(a)	B1: The constant term should be 16 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept $^4C_1$ or $\binom{4}{1}$ or 4 **as a coefficient, and** $^4C_2$ or $\binom{4}{2}$ or 6 **as another……..** Pascal's triangle may be used to establish coefficients. A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method. A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines)
(b)	B1ft: Award for correct answer as printed above or **ft their previous answer** provided it has **five terms** ft and must be **subtracting the x and x^3 terms** Allow terms in (b) to be in descending order and allow +-96x and +-216x^3 in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines)
	e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $= (16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5 **Fully correct answer with no working can score B1 in part (a) and B1 in part (b).** The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned. **Squaring the bracket and squaring again** may also earn **B1 M0 A0 A0 B1 if correct** **Omitting the final term** but otherwise correct is B1 M1 A1 A0 B0ft so 3/5 If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw)

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4.

Question number	Scheme	Marks
3 (a).	$(1+\frac{x}{4})^8 = 1 + 2x + .., \quad ......$	B1
	$+\frac{8\times7}{2}(\frac{x}{4})^2 + \frac{8\times7\times6}{2\times3}(\frac{x}{4})^3,$	M1 A1
	$= \quad +\frac{7}{4}x^2 + \frac{7}{8}x^3$ or $= \quad +1.75x^2 + 0.875x^3$	A1 (4)
(b)	States or implies that $x = 0.1$	B1
	Substitutes their value of x (provided it is <1) into series obtained in (a)	M1
	i.e. 1 + 0.2 + 0.0175 + 0.000875, = 1.2184	A1 cao (3)
		7

Alternative for (b) **Special case**	Starts again and expands $(1+0.025)^8$ to $1+\ 8\times0.025\ +\frac{8\times7}{2}(0.025)^2 + \frac{8\times7\times6}{2\times3}(0.025)^3, \ = 1.2184$ (Or 1 + 1/5 + 7/400 + 7/8000 = 1.2184)	B1,M1,A1
Notes	**(a) B1** must be simplified The **method** mark **(M1)** is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for ${}^8C_2$ and ${}^8C_3$, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs) First **A1** is for two completely correct unsimplified terms **A1** needs the fully simplified $\frac{7}{4}x^2$ *and* $\frac{7}{8}x^3$. **(b) B1** – states or uses x =0.1 or $\frac{x}{4} = \frac{1}{40}$ **M1** for substituting their value of x (0 < x <1) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but **not** 1 nor 1.025 which would earn **M0**) **A1** Should be answer printed cao (not answers which round to) and should follow correct work. **Answer with no working at all** is B0, M0, A0 States 0.1 then just writes down answer is B1 M0A0	

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5.

Question Number	Scheme		Marks
2. (a)	$\{(3+bx)^5\} = (3)^5 + \underline{{}^5C_1(3)^4(bx)} + \underline{{}^5C_2(3)^3(bx)^2} + ...$	243 as a constant term seen.	B1
	$= 243 + 405bx + 270b^2x^2 + ...$	$405bx$	B1
		$({}^5C_1 \times ... \times x)$ or $({}^5C_2 \times ... \times x^2)$	M1
		$270b^2x^2$ or $270(bx)^2$	A1 **[4]**
(b)	$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation.	M1
	So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	$b = 3$ (Ignore $b = 0$, if seen.)	A1
			[2] **6**

(a)	The terms can be "listed" rather than added. Ignore any extra terms. 1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0. 2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0. M1: For <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{5}{2}, \left(\frac{5}{2}\right), \binom{5}{1}, \left(\frac{5}{1}\right), {}^5C_2, {}^5C_1$. A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.) **<u>Alternative:</u>** Note that a factor of 3^5 can be taken out first: $3^5\left(1+\frac{bx}{3}\right)^5$, but the mark scheme still applies. **<u>Ignore subsequent working (isw)</u>**: Isw if necessary after correct working: e.g. $243 + 405bx + 270b^2x^2 + ...$ leading to $9 + 15bx + 10b^2x^2 + ...$ scores B1B1M1A1 isw. Also note that full marks could also be available in part (b), here. **<u>Special Case</u>**: Candidate writing down the first three terms in ***descending*** powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + ... = b^5x^5 + 15b^4x^4 + 90b^3x^3 + ...$ So award SC: B0B0M1A0 for either $({}^5C_4 \times ... \times x^4)$ or $({}^5C_3 \times ... \times x^3)$

(b)	M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b, <u>or</u> equating their coefficient of x to 2 times that of x^2, to get an equation in b. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0. <u>An equation in *b* alone</u> is required: e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be **Special Case** SC: M1A0 (as equation in coefficients only is not seen here). e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered). **Note:** Answer of 3 from no working scores M1A0. [redacted]

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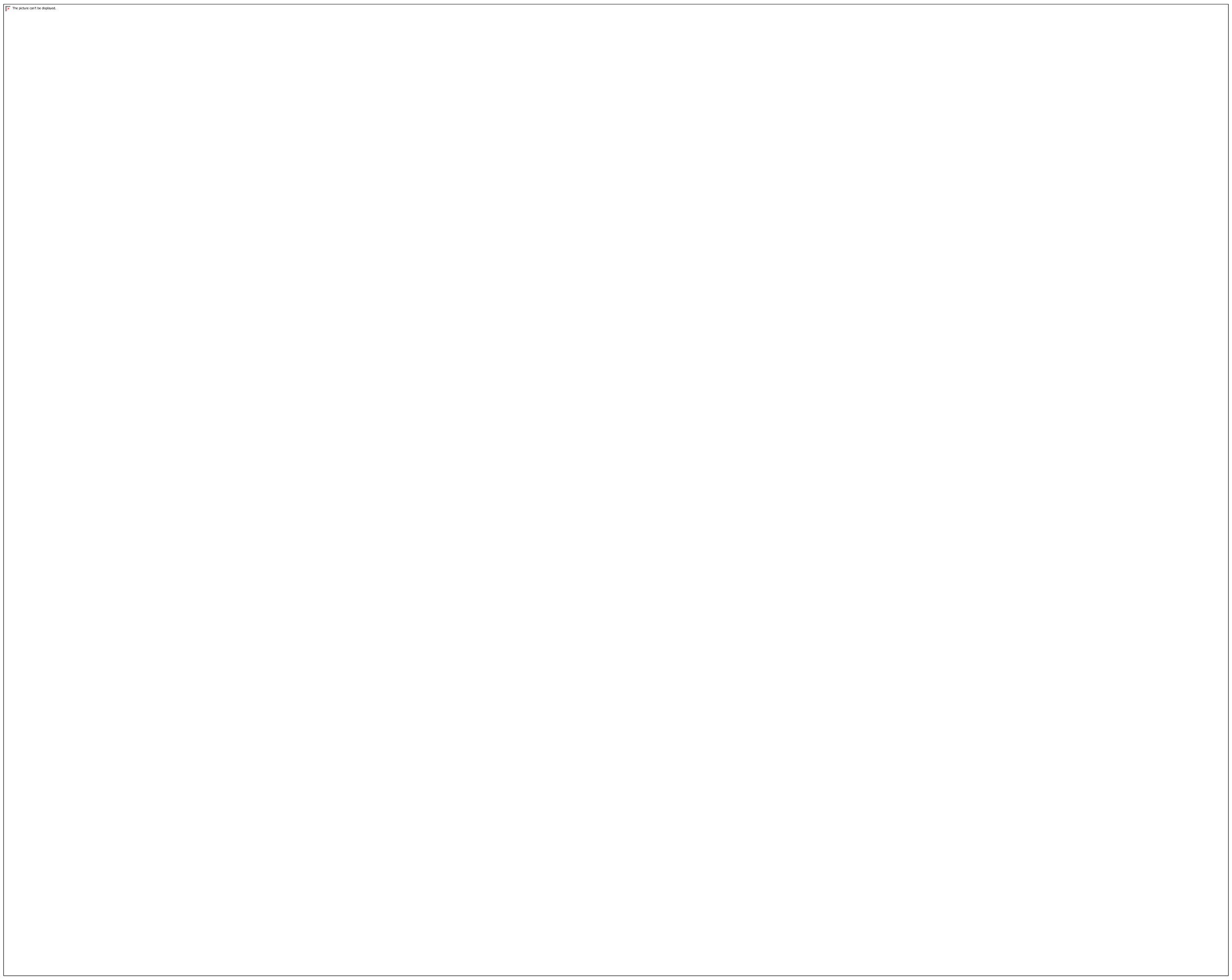

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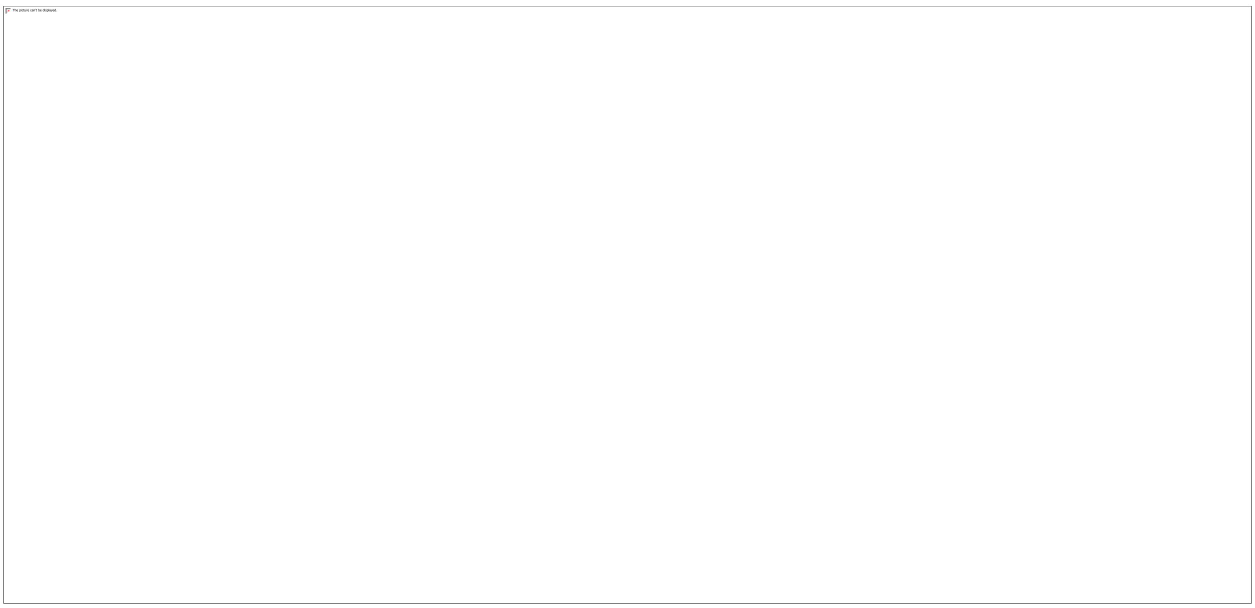

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18.

Question Number	Scheme		Marks	
Q1	(a) $(1-8x)^{\frac{1}{2}} = 1+\left(\tfrac{1}{2}\right)(-8x)+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-8x)^2+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3+\dots$		M1 A1	
	$= 1-4x-8x^2; -32x^3 - \dots$		A1; A1	(4)
	(b) $\sqrt{(1-8x)} = \sqrt{\left(1-\dfrac{8}{100}\right)}$		M1	
	$= \sqrt{\dfrac{92}{100}} = \sqrt{\dfrac{23}{25}} = \dfrac{\sqrt{23}}{5}$ *	cso	A1	(2)
	(c) $1-4x-8x^2-32x^3 = 1-4(0.01)-8(0.01)^2-32(0.01)^3$			
	$= 1-0.04-0.0008-0.000\,032 = 0.959\,168$		M1	
	$\sqrt{23} = 5\times 0.959\,168$		M1	
	$= 4.795\,84$	cao	A1	(3)
				[9]