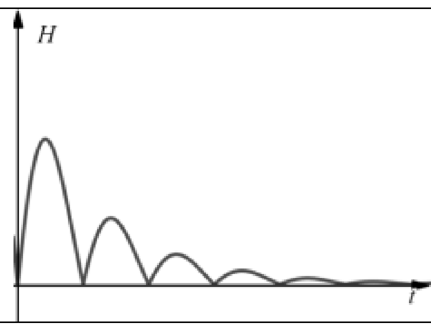
Differentiation- Mark Scheme

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
12 (a)	$\mathrm{f}(x)=10\mathrm{e}^{-0.25x}\sin x$		
	$\Rightarrow \mathrm{f}'(x)=-2.5\mathrm{e}^{-0.25x}\sin x+10\mathrm{e}^{-0.25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=0\Rightarrow -2.5\cancel{\mathrm{e}^{-0.25x}}\sin x+10\cancel{\mathrm{e}^{-0.25x}}\cos x=0$	M1	2.1
	$\frac{\sin x}{\cos x}=\frac{10}{2.5}\Rightarrow \tan x=4$ *	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops Fully correct with decreasing heights	M1 A1	1.1b 1.1b
		(2)	
(c)	Solves $\tan x=4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47)=\left\|10\mathrm{e}^{-0.25\times 4.47}\sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm ...e^{-0.25x}\sin x \pm ...e^{-0.25x}\cos x$ M1

Sight of $vdu - udv$ however is M0

A1: $f'(x) = -2.5e^{-0.25x}\sin x + 10e^{-0.25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $f'(x) = 0$, factorising/ cancelling out the $e^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x = \arctan 4$ into $f'(x)$ to score this mark.

A1*: Shows the steps $\frac{\sin x}{\cos x} = \frac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4*$. $\frac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt 76° or awrt 256° into $H(t)$

M1: Substitutes $t =$ awrt 4.47 into $H(t) = \left|10e^{-0.25t}\sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change"

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

2.

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{dy}{dx} = \frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y + \cos^2 2y = 1$ and $x = 4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx} = k\frac{1}{\cos 2y} = ..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy} = 8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

(a)

M1: Attempts to differentiate $x = 4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy} = k\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$ or $1 = k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x = 4\sin 2y \to y = ...\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx} = \frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x = 8\sin y\cos y \Rightarrow \frac{dx}{dy} = \pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx} = \frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy}=8\cos 2x \Rightarrow \frac{dy}{dx}=\frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x=8y$ oe such as $x=4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x=8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" 'both have $m=\frac{1}{8}$'

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{dx}{dy}$ and $\frac{dy}{dx}$

(c)

M1: Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$, attempts to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x. The $\frac{dy}{dx}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{dy}{dx}$ or $\frac{dx}{dy}$ Eg. $\frac{dy}{dx}=\frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{dy}{dx}=\frac{1}{2\sqrt{16-x^2}}$ The $\frac{dy}{dx}$ must be seen at least once in part (c) of this solution

....

Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x=4\sin 2y \rightarrow y=\ldots\arcsin\left(\frac{x}{4}\right) \rightarrow \frac{dy}{dx}=\frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{dy}{dx}=\frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{dy}{dx}=\frac{1}{2\sqrt{16-x^2}}$

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3.

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2}=2+6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x=4$ into their $\frac{dy}{dx}=2\times4-2-12\times4^{-\frac{1}{2}}=...$	M1	1.1b
	Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x=4$ into their $\frac{d^2y}{dx^2}=2+6\times4^{-\frac{3}{2}}=(2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2}=2.75>0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{dy}{dx}=Ax+B+Cx^{-\frac{1}{2}}$ **A1:** $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx}$ (Their $\frac{dy}{dx}$ must have a negative or fractional index)

(b)

M1: Substitutes $x=4$ into their $\frac{dy}{dx}$ and attempts to evaluate. There must be evidence $\left.\frac{dy}{dx}\right|_{x=4}=...$

Alternatively substitutes $x=4$ into an equation resulting from $\frac{dy}{dx}=0$ Eg. $\frac{36}{x}=(x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓, QED for a minimal conclusion

Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe

Alt Shows that $x=4$ is a root of the resulting equation and states "hence there is a stationary point"

All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x=4$ into their $\frac{d^2y}{dx^2}$ and calculates its value, or implies its sign by a statement such as when $x=4\Rightarrow\frac{d^2y}{dx^2}>0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x=4$ or calculates the value of y either side of $x=4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where

candidate finds $\frac{d^2y}{dx^2}$ left and right of $x=4$. Follow through on an incorrect $\frac{d^2y}{dx^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".
Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

4.

Question	Scheme	Marks	AOs
5	$\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{dy}{d\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
		(5 marks)	

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{dy}{d\theta}$ (condone it being stated as $\frac{dy}{dx}$) but tolerate slips on the coefficients and also condone $\frac{d(\sin\theta)}{d\theta}=\pm\cos\theta$ and $\frac{d(\cos\theta)}{d\theta}=\pm\sin\theta$

For quotient rule look for $\frac{dy}{d\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for
$\frac{dy}{d\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{dy}{d\theta}=...\cos\theta$

A1: A correct expression involving $\frac{dy}{d\theta}$ condoning it appearing as $\frac{dy}{dx}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow \frac{dy}{d\theta}=\frac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{dy}{d\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

5.

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx}+2y$	M1	2.1
	$2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\frac{dy}{dx}=2y-2x$	M1	2.1
	$\frac{dy}{dx}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{dy}{dx}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	

(c)	Explains that you need to solve $y = x$ and $x^2 - 2xy + 3y^2 = 50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\frac{dy}{dx}$ or $2xy \to 2x\frac{dy}{dx} + 2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\frac{dy}{dx} + 2y$ unless you see evidence that they have used the incorrect law *vu'-uv'*

A1: Fully correct derivative $2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx} = 0$

Allow attempts where candidates write $2x\text{d}x - 2x\text{d}y - 2y\text{d}x + 6y\text{d}y = 0$

but watch for students who write $\frac{dy}{dx} = 2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\frac{dy}{dx} = 2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx} = 0$

M1: For a valid attempt at making $\frac{dy}{dx}$ the subject. with two terms in $\frac{dy}{dx}$ coming from $3y^2$ and $2xy$

Look for $(\ldots \pm \ldots)\frac{dy}{dx} = \ldots\ldots$ It is implied by $\frac{dy}{dx} = \frac{2y - 2x}{6y - 2x}$

This cannot be scored from attempts such as $\frac{dy}{dx} = 2x - 2x\frac{dy}{dx} - 2y + 6y$ which only has one correct term.

A1*: $\frac{dy}{dx} = \frac{y - x}{3y - x}$ with no errors or omissions.

The previous line $\frac{dy}{dx} = \frac{2y - 2x}{6y - 2x}$ or equivalent must be seen.

(b)

M1: Deduces that $3y - x = 0$ oe

M1: Attempts to find either the *x* or *y* coordinates of *P* and *Q* by solving their $y = \frac{1}{3}x$ with $x^2 - 2xy + 3y^2 = 50$ simultaneously. Allow for finding a quadratic equation in *x* or *y* and solving to find at least one value for *x* or *y*.

This may be awarded when candidates make the numerator = 0 ie using $y = x$

A1: $\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$

dM1: Dependent upon the previous M, it is for finding the y coordinate from their x (or vice versa)
This may also be scored following the numerator being set to 0 ie using $y=x$

A1: Deduces that $P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$ OE. Allow to be $x=\ldots$ $y=\ldots$

(c)

B1ft: Explains that this is where $\frac{dy}{dx}=0$ and so you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution (or larger solution).
Allow a follow through for candidates who mix up parts (b) and (c)
Alternatively candidates could complete the square $(x-y)^2+2y^2=50$ and state that y would reach a maximum value when $x=y$ and choose the positive solution from $2y^2=50$

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6.

Question Number	Scheme		Marks
10.(a)(i)	$k=(-5)^2\times 3=75$	M1: Attempts to find the 'y' intercept. Accept as evidence $(-5)^2\times 3$ with or without the bracket. If they expand f(x) to polynomial form here **then they must then select their constant to score this mark**. May be implied by sight of 75 on the diagram.	M1A1
		A1: $k=75$. Must clearly be identified as k. Allow this mark even from an incorrect or incomplete expansion as long as the constant $k=75$ is obtained. Do not isw e.g. if 75 is seen followed by $k=-75$ score M1A0.	
(ii)	$c=\frac{5}{2}$ only	$c=\frac{5}{2}$ oe (and no other values). Do not award just from the diagram – must be stated as the value of c.	B1
			(3)
(b)	$f(x)=(2x-5)^2(x+3)=(4x^2-20x+25)(x+3)=4x^3-8x^2-35x+75$ Attempts f(x) as a cubic polynomial by attempting to square the first bracket and multiply by the linear bracket or expands $(2x-5)(x+3)$ and then multiplies by $2x-5$ **Must be seen or used in (b) but may be done in part (a).** Allow poor squaring e.g. $(2x-5)^2=4x^2\pm 25$		M1

	$(\text{f}'(x)=)12x^2-16x-35$*	M1: Reduces powers by 1 in **all** terms including any constant $\to 0$ A1: Correct proof. Withhold this mark if there have been any errors including missing brackets earlier e.g. $(2x-5)^2(x+3)=4x^2-20x+25(x+3)=...$	M1A1*
			(3)
(c)	$\text{f}'(3)=12\times3^2-16\times3-35$	Substitutes $x=3$ into **their** $\text{f}'(x)$ **or the given** $\text{f}'(x)$. Must be a changed function i.e. not into f(x).	M1
	$12x^2-16x-35='25'$	Sets **their** $\text{f}'(x)$ **or the given** $\text{f}'(x)=$ their $\text{f}'(3)$ with a consistent f'. **Dependent on the previous method mark.**	**d**M1
	$12x^2-16x-60=0$	$12x^2-16x-60=0$ or equivalent 3 term quadratic e.g. $12x^2-16x=60$. (A correct quadratic equation may be implied by later work). **This is cso so must come from correct work – i.e. they must be using the <u>given</u>** $\text{f}'(x)$.	A1 cso
	$(x-3)(12x+20)=0\Rightarrow x=...$	Solves 3 term quadratic by suitable method – see General Principles. **Dependent on both previous method marks.**	**dd**M1
	$x=-\frac{5}{3}$	$x=-\frac{5}{3}$ oe clearly identified. If $x=3$ is also given and not rejected, this mark is withheld. (allow -1.6 recurring as long as it is clear i.e. a dot above the 6). **This is cso and must come from correct work – i.e. they must be using the <u>given</u>** $\text{f}'(x)$.	A1 cso
			(5)
			(11 marks)

Alt (b) Product rule.	$\text{f}(x)=(2x-5)^2(x+3)\Rightarrow\text{f}'(x)=(2x-5)^2\times1+(x+3)\times4(2x-5)$ M1: Attempts product rule to give an expression of the form $p(2x-5)^2+q(x+3)(2x-5)$ M1: Multiplies out and collects terms A1: $\text{f}'(x)=12x^2-16x-35$*	M1 M1A1*

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7.

Question Number	Scheme	Notes	Marks
7.	$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}$		
	$\frac{2x^3-7}{3\sqrt{x}} = \frac{2x^3}{3\sqrt{x}} - \frac{7}{3\sqrt{x}} = \frac{2}{3}x^{\frac{5}{2}} - \frac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\frac{2x^3-7}{3\sqrt{x}} = 2x^3 - 7 + 3x^{-\frac{1}{2}}$	M1
	$x^n \to x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\frac{dy}{dx} =\right) 6x + 2x^{-\frac{2}{3}} + \frac{5}{3}x^{\frac{3}{2}} + \frac{7}{6}x^{-\frac{3}{2}}$	A1: $6x$. Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw. A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\frac{2}{1}x^{-\frac{2}{3}}$ but allow $\frac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw. A1: $\frac{5}{3}x^{\frac{3}{2}}$. Must be simplified but allow e.g. $1\frac{2}{3}x^{1.5}$ or e.g. $\frac{5}{3}\sqrt{x^3}$. Award when first seen and isw. A1: $\frac{7}{6}x^{-\frac{3}{2}}$. Must be simplified but allow e.g. $1\frac{1}{6}x^{-1\frac{1}{2}}$ or e.g. $\frac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	A1A1A1A1
	In an otherwise fully correct solution, penalise the presence of + c by deducting the final A1		
			[6]
	Use of Quotient Rule: First M1 and final A1A1 (Other marks as above)		
	$\frac{d\left(\frac{2x^3-7}{3\sqrt{x}}\right)}{dx} = \frac{3\sqrt{x}(6x^2) - (2x^3-7)\frac{3}{2}x^{-\frac{1}{2}}}{(3\sqrt{x})^2}$	Uses **correct** quotient rule	M1
	$= \frac{10x^{\frac{5}{2}} + 7x^{-\frac{1}{2}}}{6x}$	A1: Correct first term of numerator and correct denominator A1: All correct as simplified as shown	A1A1
	So $\frac{dy}{dx} = 6x + 2x^{-\frac{2}{3}} + \frac{10x^{\frac{5}{2}} + 7x^{-\frac{1}{2}}}{6x}$ scores full marks		
			6 marks

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8. N

Question Number	Scheme		Marks
8. (a)	$\{V =\}\ 2x^2y = 81$	$2x^2y = 81$	B1 oe
	$\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$		
	$y = \dfrac{81}{2x^2} \Rightarrow L = 12x + 4\left(\dfrac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L = 12x + \dfrac{162}{x^2}$ **AG**	Correct solution only. **AG.**	A1 **cso**
			[3]
(b)	$\dfrac{dL}{dx} = 12 - \dfrac{324}{x^3}\ \{= 12 - 324x^{-3}\}$	Either $12x \to 12$ or $\dfrac{162}{x^2} \to \dfrac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\dfrac{dL}{dx} =\right\} 12 - \dfrac{324}{x^3} = 0 \Rightarrow x^3 = \dfrac{324}{12}; = 27 \Rightarrow x = 3$	$L' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value"	M1;
		$x = \sqrt[3]{27}$ or $x = 3$	A1 **cso**
	$\{x = 3,\}\ L = 12(3) + \dfrac{162}{3^2} = 54$ (cm)	Substitute candidate's value of $x\ (\neq 0)$ into a formula for L.	ddM1
		54	A1 **cao**
			[6]
(c)	$\{\text{For } x = 3\},\ \dfrac{d^2L}{dx^2} = \dfrac{972}{x^4} > 0 \Rightarrow$ Minimum	Correct ft L'' and considering sign.	M1
		$\dfrac{972}{x^4}$ and > 0 and conclusion.	A1 **[2]**
			11

(a)	B1: For any correct form of $2x^2y = 81$. (may be unsimplified). Note that $2x^3 = 81$ is B0. Otherwise, candidates can use any symbol or letter in place of y. M1: Making y the subject of their formula and substituting this into a correct expression for L. A1: Correct solution only. Note that the answer is given.
(b)	**Note you can mark parts (b) and (c) together.** 2nd M1: Setting their $\frac{dL}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". The power of x must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of x or L from their x without inequalities. $L' = 0$ can be implied by $12 = \frac{324}{x^3}$. 2nd A1: $x^3 = 27 \Rightarrow x = \pm 3$ scores A0. 2nd A1: can be given for no value of x given but followed through by correct working leading to $L = 54$. 3rd M1: Note that this method mark is dependent upon the two previous method marks being awarded.
(c)	M1: for attempting correct ft second derivative and considering its sign. A1: Correct second derivative of $\frac{972}{x^4}$ (need not be simplified) and a valid reason (e.g. > 0), and conclusion. Need to conclude minimum (allow x and not L is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their L and not x into L'' is A0. **Note:** 2 marks can be scored from a wrong value of x, no value of x found or from not substituting in the value of their x into L''. Gradient test or testing values either side of their x scores M0A0 in part (c). **Throughout this question allow confused notation such as** $\frac{dy}{dx}$ for $\frac{dL}{dx}$.

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9.

Question Number	Scheme	Marks
7(i) (a)	$y=2x\left(x^2-1\right)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^5\times 2+2x\times 10x\left(x^2-1\right)^4$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4\left(2x^2-2+20x^2\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$	M1 A1
		(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}\geqslant 0\Rightarrow\left(22x^2-2\right)\geqslant 0\Rightarrow$ critical values of $\pm\frac{1}{\sqrt{11}}$	M1
	$x\geqslant\frac{1}{\sqrt{11}}\quad x\leqslant-\frac{1}{\sqrt{11}}$	A1
		(2)
(ii)	$x=\ln(\sec 2y)\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$	B1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\tan 2y}=\frac{1}{2\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	M1 M1 A1
		(4)
		10 marks

Alt 1 (ii)	$x=\ln(\sec 2y)\Rightarrow\sec 2y=\mathrm{e}^x$	
	$\Rightarrow 2\sec 2y\tan 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$	B1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^x}{2\sec 2y\tan 2y}=\frac{\mathrm{e}^x}{2\mathrm{e}^x\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	M1M1A1
		(4)
Alt 2 (ii)	$y=\frac{1}{2}\arccos\left(\mathrm{e}^{-x}\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}\times\frac{1}{\sqrt{1-\left(\mathrm{e}^{-x}\right)^2}}\times-\mathrm{e}^{-x}$	B1M1M1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	A1
		(4)

(i)(a)

M1 Attempts the product rule to differentiate $2x(x^2-1)^5$ to a form $A\left(x^2-1\right)^5 + Bx^n\left(x^2-1\right)^4$ where $n=1$ or 2. and A, B >0 If the rule is stated it must be correct, and not with a "$-$" sign.

A1 Any unsimplified but correct form $\left(\frac{dy}{dx}\right) = 2\left(x^2-1\right)^5 + 20x^2\left(x^2-1\right)^4$

M1 For taking a common factor of $\left(x^2-1\right)^4$ out of a suitable expression

Look for $A\left(x^2-1\right)^5 \pm Bx^n\left(x^2-1\right)^4 = \left(x^2-1\right)^4\left\{A\left(x^2-1\right) \pm Bx^n\right\}$ but you may condone missing brackets

It can be scored from a vu'-uv' or similar.

A1 $\left(\frac{dy}{dx}\right) = \left(x^2-1\right)^4\left(22x^2-2\right)$ Expect $g(x)$ to be simplified but accept $\frac{dy}{dx} = \left(x^2-1\right)^4 2\left(11x^2-1\right)$

There is no need to state g(x) and remember to isw after a correct answer. This must be in part (a).

(i)(b)

M1 Sets their $\frac{dy}{dx} \geq 0, > 0$ or $\frac{dy}{dx} = 0$ and proceeds to find one of the critical values for **their** $g(x)$ or their $\frac{dy}{dx} = 0$ rearranged and $\div\left(x^2-1\right)^4$ if g(x) not found. g(x) should be at least a 2TQ with real roots. If g(x) is factorised, the usual rules apply. The M cannot be awarded from work **just** on $\left(x^2-1\right)^4 \geq 0$ ie $x = \pm 1$

You may see and accept decimals for the M.

A1 cao $x \geq \frac{1}{\sqrt{11}}$ $x \leq -\frac{1}{\sqrt{11}}$ or exact equivalent only. Condone $x \geq \frac{1}{\sqrt{11}}$ $x \leq -\frac{1}{\sqrt{11}}$, with $x \geq 1, x \leq -1$

Accept exact equivalents such as $x \geq \frac{\sqrt{11}}{11}$ $x \leq -\frac{\sqrt{11}}{11}$; $|x| \geq \frac{1}{\sqrt{11}}$; $\left\{\left(-\infty, -\frac{\sqrt{11}}{11}\right] \cup \left[\frac{\sqrt{11}}{11}, \infty\right)\right\}$

Condone the word "and" appearing between the two sets of values.

Withhold the final mark if $x \geq \frac{1}{\sqrt{11}}$ $x \leq -\frac{1}{\sqrt{11}}$, appears with values not in this region eg $x \leq 1, x \geq -1$

(ii)

B1 Differentiates and achieves a correct line involving $\frac{dy}{dx}$ or $\frac{dx}{dy}$

Accept $\frac{dx}{dy} = \frac{1}{\sec 2y} \times 2\sec 2y \tan 2y$, $\frac{dx}{dy} = -\frac{1}{\cos 2y} \times -2\sin 2y$ $2\sec 2y \tan 2y \frac{dy}{dx} = e^x$

M1 For inverting their expression for $\frac{dx}{dy}$ to achieve an expression for $\frac{dy}{dx}$.

The variables (on the rhs) must be consistent, you may condone slips on the coefficients but not the terms.
In the alternative method it is for correctly changing the subject

M1 Scored for using $\tan^2 2y = \pm 1 \pm \sec^2 2y$ **and** $\sec 2y = e^x$ to achieve $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x

Alternatively they could use $\sin^2 2y + \cos^2 2y = 1$ with $\cos 2y = e^{-x}$ to achieve $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x

For the M mark you may condone $\sec^2 2y = \left(e^x\right)^2$ appearing as e^{x^2}

A1 cso $\frac{dy}{dx}=\frac{1}{2\sqrt{e^{2x}-1}}$ Final answer, do not allow if students then simplify this to eg. $\frac{dy}{dx}=\frac{1}{2e^{x}-1}$

Condone $\frac{dy}{dx}=\pm\frac{1}{2\sqrt{e^{2x}-1}}$ but do not allow $\frac{dy}{dx}=-\frac{1}{2\sqrt{e^{2x}-1}}$

Allow a misread on $x=\ln(\sec y)$ for the two method marks only

10.

Question Number	Scheme	Marks
8 (a)	$P_0=\frac{100}{1+3}+40=65$	B1
		(1)
	$\frac{d}{dt}e^{kt}=Ce^{kt}$	M1
(b)	$\frac{dP}{dt}=\frac{(1+3e^{-0.9t})\times -10e^{-0.1t}-100e^{-0.1t}\times -2.7e^{-0.9t}}{(1+3e^{-0.9t})^2}$	M1 A1
		(3)
(c)(i)	At maximum $-10e^{-0.1t}-30e^{-0.1t}\times e^{-0.9t}+270e^{-0.1t}\times e^{-0.9t}=0$	
	$e^{-0.1t}(-10+240e^{-0.9t})=0$	
	$e^{-0.9t}=\frac{10}{240}$ oe $e^{0.9t}=24$	M1
	$-0.9t=\ln\left(\frac{1}{24}\right)\Rightarrow t=\frac{10}{9}\ln(24)=3.53$	M1, A1
(c) (ii)	Sub $t=3.53\Rightarrow P_T=102$	A1
		(4)
(d)	40	B1
		(1)
		9 marks

(a)

B1 $\left(P_0 =\right)65$

(b)

M1 For sight of $\dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$ (Allow C =1)This may be within an incorrect product or quotient rule

M1 Scored for a full application of the quotient rule. If the formula is quoted it should be correct.

The denominator should be present even when the correct formula has been quoted.

In cases where a formula has not been quoted it is very difficult to judge that a correct formula has been used (due to the signs between the terms). So…………….

if the formula has not been quoted look for the **order** of the terms $\dfrac{(1+3\mathrm{e}^{-0.9t})\times p\mathrm{e}^{-0.1t} - q\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t}}{(1+3\mathrm{e}^{-0.9t})^2}$ $\dfrac{(1+3\mathrm{e}^{-0.9t})\times p\mathrm{e}^{-0.1t} + q\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t}}{(1+3\mathrm{e}^{-0.9t})^2}$

For the product rule. Look for $a\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-1} \pm b\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-2}$ either way around

Penalise if an incorrect formula is quoted . Condone missing brackets in both cases.

A1 A correct **unsimplified** answer.

Eg using quotient rule $\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}\right) = \dfrac{-10\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)+270\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$ oe $\dfrac{-10\mathrm{e}^{-0.1t}+240\mathrm{e}^{-1t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$ simplified

Eg using product rule $\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}\right) = -10\mathrm{e}^{-0.1t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-1} + 270\mathrm{e}^{-0.1t}\mathrm{e}^{-0.9t}\left(1+3\mathrm{e}^{-0.9t}\right)^{-2}$ oe

Remember to isw after a correct (unsimplified) answer.

There is no need to have the $\dfrac{\mathrm{d}P}{\mathrm{d}t}$ and it could be called $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(c)(i) Do NOT allow any marks in here without sight/implication of $\dfrac{\mathrm{d}P}{\mathrm{d}t}=0, \dfrac{\mathrm{d}P}{\mathrm{d}t}<0$ OR $\dfrac{\mathrm{d}P}{\mathrm{d}t}>0$

The question requires the candidate to find t using part (b) so it is possible to do this part using inequalities using the same criteria as we apply for the equality. All marks in (c) can be scored from an incorrect denominator (most likely v), no denominator, or using a numerator the wrong way around ie $uv'-u'v$

M1 Sets their $\dfrac{\mathrm{d}P}{\mathrm{d}t}=0$ or the numerator of their $\dfrac{\mathrm{d}P}{\mathrm{d}t}=0$, factorises out or cancels a term in $\mathrm{e}^{-0.1t}$ to reach a form $A\mathrm{e}^{\pm 0.9t} = B$ oe. Alternatively they could combine terms to reach $A\mathrm{e}^{-t} = B\mathrm{e}^{-0.1t}$ or equivalent

Condone a double error on $\mathrm{e}^{-0.1t}\times\mathrm{e}^{-0.9t} = \mathrm{e}^{-0.1t\times -0.9t}$ or similar before factorising. **Look for correct indices.**

If they use the product rule then expect to see their $\dfrac{\mathrm{d}P}{\mathrm{d}t}=0$ followed by multiplication of $\left(1+3\mathrm{e}^{-0.9t}\right)^2$ before similar work to the quotient rule leads to a form $A\mathrm{e}^{\pm 0.9t} = B$

M1 Having set the numerator of their $\frac{\text{ω}}{\text{d}t}=0$ and obtained either $\text{e}^{\pm kt}=C$ (k may be incorrect) or $Ae^{-t}=Be^{-0.1t}$ it is awarded for the correct order of operations, taking ln's leading to $t=..$

It cannot be awarded from impossible equations Eg $\text{e}^{\pm 0.9t}=-0.3$

A1 cso $t=$ awrt 3.53 Accept $t=\frac{10}{9}\ln(24)$ or exact equivalent.

(c)(ii)

A1 awrt 102 following 3.53 The M's must have been awarded. This is not a B mark.

(d)

B1 Sight of 40

Condone statements such as $P\to 40$ $k\ldots 40$ or likewise

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11.

Question	Scheme	Marks
2(a)	$y=\frac{4x}{(x^2+5)}\Rightarrow\left(\frac{\text{d}y}{\text{d}x}\right)=\frac{4(x^2+5)-4x\times 2x}{(x^2+5)^2}$	M1A1
	$\Rightarrow\left(\frac{\text{d}y}{\text{d}x}\right)=\frac{20-4x^2}{(x^2+5)^2}$	M1A1
		(4)
(b)	$\frac{20-4x^2}{(x^2+5)^2}<0\Rightarrow x^2>\frac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5}, x>\sqrt{5}$ or equivalent	dM1A1
		(3)
		7 marks

(a)M1 Attempt to use the **quotient rule** $\frac{vu'-uv'}{v^2}$ with $u=4x$ and $v=x^2+5$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,$ $v=x^2+5, v'=Bx$ followed by their $\frac{vu'-uv'}{v^2}$

If the rule is neither quoted nor implied only accept expressions of the form

$\frac{A(x^2+5)-4x\times Bx}{(x^2+5)^2}, A,B>0$ You may condone missing (invisible) brackets

Alternatively uses the **product rule** with $u(/v)=4x$ and $v(/u)=(x^2+5)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,$ $v=x^2+5, v'=Bx(x^2+5)^{-2}$ followed by their $vu'+uv'$. If the rule is neither quoted nor implied only accept expressions of the form

$A(x^2+5)^{-1}\pm 4x\times Bx(x^2+5)^{-2}$

A1 f '(x) correct (unsimplified). For the product rule look for versions of $4(x^2+5)^{-1} - 4x \times 2x(x^2+5)^{-2}$

M1 Simplifies to the form $f'(x) = \dfrac{A+Bx^2}{(x^2+5)^2}$ oe. This is not dependent so could be scored from $\dfrac{v'u-u'v}{v^2}$

When the product rule has been used the A of $A(x^2+5)^{-1}$ must be adapted.

A1 CAO. Accept exact equivalents such as $(f'(x)) = \dfrac{4(5-x^2)}{(x^2+5)^2}$, $-\dfrac{4x^2-20}{(x^2+5)^2}$ or $\dfrac{-4(x^2-5)}{x^4+10x^2+25}$

Remember to isw after a correct answer

(b)

M1 Sets their numerator either $=0$, <0, $\leqslant 0$ >0, $\geqslant 0$ and proceeds to at least **one** value for x

For example $20-4x^2 \geqslant 0 \Rightarrow x \geqslant \sqrt{5}$ will be M1 dM0 A0.

It cannot be scored from a numerator such as 4 or indeed $20+4x^2$

dM1 Achieves **two** critical values for their numerator $=0$ and chooses the outside region

Look for $x <$ smaller root, $x >$ bigger root. Allow decimals for the roots.

Condone $x \leqslant -\sqrt{5}, x \geqslant \sqrt{5}$ and expressions like $-\sqrt{5} > x > \sqrt{5}$

If they have $4x^2-20<0$ following an incorrect derivative they should be choosing the inside region

A1 Allow $x < -\sqrt{5}, x > \sqrt{5}$ $x < -\sqrt{5}$ or $x > \sqrt{5}$ $\{x : -\infty < x < -\sqrt{5} \cup \sqrt{5} < x < \infty\}$ $|x| > \sqrt{5}$

Do not allow for the A1 $x < -\sqrt{5}$ and $x > \sqrt{5}$. $\sqrt{5} < x < -\sqrt{5}$ or $\{x : -\infty < x < -\sqrt{5} \cap \sqrt{5} < x < \infty\}$

but you may isw following a correct answer.

12.

Question	Scheme	Marks
5 (i)	$y = e^{3x}\cos 4x \Rightarrow \left(\dfrac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \dfrac{1}{4}\arctan\dfrac{3}{4}$	M1
	$\Rightarrow x =$ awrt 0.9463 4dp	A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \dfrac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\dfrac{dx}{dy} = 2\sin 4y \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sin 4y} = \dfrac{1}{2}\text{cosec}\,4y$	M1A1
		(5) **(10 marks)**

(ii) Alt I	$x=\sin^2 2y \Rightarrow x=\frac{1}{2}-\frac{1}{2}\cos 4y$ $\frac{dx}{dy}=2\sin 4y$ $\Rightarrow \frac{dy}{dx}=\frac{1}{2\sin 4y}=\frac{1}{2}\text{cosec}4y$	2nd M1 1st M1 A1 M1A1 **(5)**
(ii) Alt II	$x^{\frac{1}{2}}=\sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}}=2\cos 2y\frac{dy}{dx}$ Uses $x^{\frac{1}{2}}=\sin 2y$ AND $\sin 4y=2\sin 2y\cos 2y$ in their expression $\frac{dy}{dx}=\frac{1}{2\sin 4y}=\frac{1}{2}\text{cosec}4y$	M1A1 M1 M1A1 **(5)**
(ii) Alt III	$x^{\frac{1}{2}}=\sin 2y \Rightarrow 2y=\text{invsin}\,x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx}=\frac{1}{\sqrt{1-x}}\times\frac{1}{2}x^{-\frac{1}{2}}$ Uses $x^{\frac{1}{2}}=\sin 2y$, $\sqrt{1-x}=\cos 2y$ and $\sin 4y=2\sin 2y\cos 2y$ in their expression $\Rightarrow \frac{dy}{dx}=\frac{1}{2\sin 4y}=\frac{1}{2}\text{cosec}4y$	M1A1 M1 M1A1 **(5)**

(i)

M1 Uses the product rule $uv'+vu'$ to achieve $\left(\frac{dy}{dx}\right)=Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A,B\neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx}=\cos 4x\times 3e^{3x}+e^{3x}\times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv\tan 4x$, moves from $\tan 4x=C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.
If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$
Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x=$ awrt 0.9463 .
Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.
There is no need for lhs to be seen/ correct
If the product rule is used look for $\cancel{\frac{dy}{dx}} = \pm A\sin 2y\cos 2y \pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy} = 2\sin 2y \times 2\cos 2y = (4\sin 2y\cos 2y)$ or
$1 = 2\sin 2y \times 2\cos 2y \frac{dy}{dx}$

M1 Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression.
You may just see a statement such as $4\sin 2y\cos 2y = 2\sin 4y$ which is fine.
Candidates who write $\cancel{\frac{dy}{dx}} = A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dy}{dx}} = \frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx} = 1 \Big/ \frac{dx}{dy}$ **for their expression in *y***. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = 2\text{cosec}4y$ is condoned for the M1
If $\frac{dx}{dy} = a + b$ do not allow $\frac{dy}{dx} = \frac{1}{a} + \frac{1}{b}$

A1 $\frac{dy}{dx} = \frac{1}{2}\text{cosec}4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x = \sin^2 2y \Rightarrow x = \pm\frac{1}{2} \pm \frac{1}{2}\cos 4y$ from $\cos 4y = \pm 1 \pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}} = \sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}} = Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y = \text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}} = N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}} \times x^{-0.5}$

13.

Question	Scheme	Marks
6(a)	$x^2+x-6\overline{\smash{)}x^4+x^3-3x^2+7x-6}$ with quotient x^2+3	
	$\underline{x^4+x^3-6x^2}$	
	$3x^2+7x-6$	M1 A1
	$\underline{3x^2+3x-18}$	
	$4x+12$	
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
(b)	$\text{f}'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $\text{f}'(x=3)=2\times 3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{\text{f}'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3,\text{f}(3))=(3,16)$ to form eqn of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5)
		(9 marks)

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$\begin{array}{r} x^2(+..x)+A \\ x^2+x-6\overline{)x^4+x^3-3x^2+7x-6} \\ \underline{x^4+x^3-6x^2} \\ \\ \overline{(Cx)+D} \end{array}$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $(x^3-2x^2+3x-2)\div(x-2)=x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient = x^2+3 and Remainder = $4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right)\equiv \text{Their Quadratic Quotient} + \frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\dfrac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\dfrac{B}{x-2}\to 2x\pm\dfrac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\dfrac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\dfrac{Rx+S}{x\pm T}$ using the quotient rule oe.

A1ft $x^2+A+\dfrac{B}{x-2}\to 2x-\dfrac{B}{(x-2)^2}$ oe. FT on their numerical A, B for for $x^2+A+\dfrac{B}{x-2}$ only

M1 Subs $x=3$ into their $f'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\dfrac{1}{\text{their } f'(3)}$ and the point must be $(3, f(3))$. Don't be overly concerned about how they found their f(3), ie accept x=3 y =.

Look for $y-f(3)=-\dfrac{1}{f'(3)}(x-3)$ or $(y-f(3))\times -f'(3)=(x-3)$

If the form $y=mx+c$ is used they must proceed as far as $c=$

A1 cso $y-16=-\dfrac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x+3)$ Compare 2 terms (or substitute 2 values) AND solve simultaneously ie $x^2 \Rightarrow A-6=-3, \quad x \Rightarrow A+B=7, \quad \text{const} \Rightarrow -6A+3B=-6$ $A=3, B=4$	M1 M1 A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve
$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x\pm3)$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of ***A*** and ***B***.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

Alt (b) is attempted by the quotient (or product rule)

ALT (b) 1st 3 marks	$f'(x)=\dfrac{(x^2+x-6)(4x^3+3x^2-6x+7)-(x^4+x^3-3x^2+7x-6)(2x+1)}{(x^2+x-6)^2}$ Subs $x=3$ into	M1A1 M1

M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and achieves an expression of the form $f'(x)=\dfrac{(x^2+x-6)(..x^3........)-(x^4+x^3-3x^2+7x-6)(..x..)}{(x^2+x-6)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=(x^2+x-6)^{-1}$

Note that this can score full marks from a partially solved part (a) where $f(x)\equiv x^2+3+\dfrac{4x+12}{x^2+x-6}$

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14.

Question Number	Scheme	Marks
5.(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x=(4y-\sin 2y)^2 \Rightarrow \frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\frac{\pi}{2}$ into $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow \frac{dx}{dy}=24\pi \quad (=75.4)$ / $\frac{dy}{dx}=\frac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$	M1
	Using $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$ with $x=0 \Rightarrow y=\frac{\pi}{3}$ cso	M1, A1
		(6)
		(7 marks)
Alt (b) I	$x=(4y-\sin 2y)^2 \Rightarrow x^{0.5}=4y-\sin 2y$ $\Rightarrow 0.5x^{-0.5}\frac{dx}{dy}=4-2\cos 2y$	M1A1
Alt (b) II	$x=(16y^2-8y\sin 2y+\sin^2 2y)$ $\Rightarrow 1=32y\frac{dy}{dx}-8\sin 2y\frac{dy}{dx}-16y\cos 2y\frac{dy}{dx}+4\sin 2y\cos 2y\frac{dy}{dx}$ Or $1dx=32y\,dy-8\sin 2y\,dy-16y\cos 2y\,dy+4\sin 2y\cos 2y\,dy$	M1A1

(a)

B1 $\quad p = 4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x = 4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form

$A(4y-\sin 2y)(B \pm C\cos 2y),\quad A,B,C \neq 0$ on the right hand side

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x=\left(16y^2-8y\sin 2y+\sin^2 2y\right)\Rightarrow\frac{dx}{dy}=Py\pm Q\sin 2y\pm Ry\cos 2y\pm S\sin 2y\cos 2y\quad P,Q,R,S\neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}...=4-Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$ or $\frac{dy}{dx}=\frac{1}{2(4y-\sin 2y)(4-2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.

In the alternative $\frac{dx}{dy}=32y-8\sin 2y-16y\cos 2y+4\sin 2y\cos 2y$

M1 Sub $y=\frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y=\frac{\pi}{2}\Rightarrow\frac{dx}{dy}=...$

It is not dependent upon the previous M1 but it must be a changed $x=(4y-\sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2',\frac{\pi}{2}\right)$.

Allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical }\left(\frac{dx}{dy}\right)}\left(x-\text{their }4\pi^2\right)$

Allow for $\left(y-\frac{\pi}{2}\right)\times\text{their numerical }\left(\frac{dx}{dy}\right)=\left(x-\text{their }4\pi^2\right)$

Even allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical }\left(\frac{dx}{dy}\right)}(x-p)$

It is possible to score this by stating the equation $y=\frac{1}{24\pi}x+c$ as long as $\left('4\pi^2',\frac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y=mx+c$ and stating the value of 'c'

Or setting $x=0$ in their $y-\frac{\pi}{2}=\frac{1}{24\pi}\left(x-4\pi^2\right)$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\frac{\frac{\pi}{2}-y}{4\pi^2}=\frac{1}{24\pi}\Rightarrow y=..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y=\frac{\pi}{3}$. You do not have to see $\left(0,\frac{\pi}{3}\right)$

15.

Question Number	Scheme	Marks
9.(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow f'(x)=\dfrac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1, A1
	$\Rightarrow f'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $f'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow$ f(x) is an increasing function as $f'(x)>0$,	M1
	$f'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)

(a)

B1 For seeing $x^2-3kx+2k^2=(x-2k)(x-k)$ anywhere in the solution

M1 For writing as a single term or two terms with the same denominator

Score for $2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$ or

$$2-\frac{(x-5k)(x-k)}{(x-2k)(x-k)}=\frac{2(x-2k)(x-k)-(x-5k)(x-k)}{(x-2k)(x-k)} \qquad \left(=\frac{x^2-k^2}{x^2-3kx+2k^2}\right)$$

A1* Proceeds without any errors (including bracketing) to $=\dfrac{x+k}{(x-2k)}$

(b)

M1 Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$.

If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=x-2k$ with their u',v' *and* $\frac{vu'-uv'}{v^2}$

If it is neither stated nor implied only accept expressions of the form $\mathrm{f}'(x)=\frac{x-2k-x\pm k}{(x-2k)^2}$

The mark can be scored for applying the product rule to $y=(x+k)(x-2k)^{-1}$ If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=(x-2k)^{-1}$ with their u',v' and $vu'+uv'$

If it is neither stated nor implied only accept expressions of the form $\mathrm{f}'(x)=(x-2k)^{-1}\pm(x+k)(x-2k)^{-2}$

Alternatively writes $y=\frac{x+k}{x-2k}$ as $y=1+\frac{3k}{x-2k}$ and differentiates to $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{A}{(x-2k)^2}$

A1 Any correct form (unsimplified) form of $\mathrm{f}'(x)$.

$\mathrm{f}'(x)=\frac{(x-2k)\times1-(x+k)\times1}{(x-2k)^2}$ by quotient rule

$\mathrm{f}'(x)=(x-2k)^{-1}-(x+k)(x-2k)^{-2}$ by product rule

and $\mathrm{f}'(x)=\frac{-3k}{(x-2k)^2}$ by the third method

A1 cao $\mathrm{f}'(x)=\frac{-3k}{(x-2k)^2}$. Allow $\mathrm{f}'(x)=\frac{-3k}{x^2-4kx+4k^2}$

As this answer is not given candidates you may allow recovery from missing brackets

(c) Note that this is B1 B1 on e pen. We are scoring it M1 A1

M1 If in part (b) $\mathrm{f}'(x)=\frac{-Ck}{(x-2k)^2}$, look for f($x$) is an increasing function as $\mathrm{f}'(x)$ / gradient >0

Accept a version that states as $k<0\Rightarrow -Ck>0$ hence increasing

If in part (b) $\mathrm{f}'(x)=\frac{(+)Ck}{(x-2k)^2}$, look for f($x$) is an decreasing function as $\mathrm{f}'(x)$ / gradient <0

Similarly accept a version that states as $k<0\Rightarrow (+)Ck<0$ hence decreasing

A1 Must have $\mathrm{f}'(x)=\frac{-3k}{(x-2k)^2}$ and give a reason that links the gradient with its sign.

There must have been reference to the sign of both numerator and denominator to justify the overall positive sign.

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16.

Question Number	Scheme	Marks
1.(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2},\quad x>2$	
	Applies $\dfrac{vu'-uv'}{v^2}$ to get $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	M1A1
	$=\dfrac{-9}{(x-2)^2}$	A1*
		(3)
(b)	$\dfrac{-9}{(x-2)^2}=-1\Rightarrow x=..$	M1
	(5,7)	A1,A1
		(3)
		6 marks
Alt 1.(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2}=4+\dfrac{9}{x-2}$	
	Applies chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$	M1
	$=-9(x-2)^{-2}=\dfrac{-9}{(x-2)^2}$	A1, A1*
		(3)

(a)

M1 Applies the quotient rule to $\mathrm{f}(x)=\frac{4x+1}{x-2}$ with $u=4x+1$ and $v=x-2$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=x-2, u'=.., v'=..$ followed by $\frac{vu'-uv'}{v^2}$.

If neither quoted nor implied only accept expressions of the form $\frac{(x-2)\times A-(4x+1)\times B}{(x-2)^2}$ $A, B>0$ allowing for a sign slip inside the brackets.

Condone missing brackets for the method mark but not the final answer mark.

Alternatively they could apply the product rule with $u=4x+1$ and $v=(x-2)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=(x-2)^{-1}, u'=.., v'=..$ followed by $vu'+uv'$.

If it is neither quoted nor implied only accept expressions of the form/ or equivalent to the form

$$(x-2)^{-1}\times C+(4x+1)\times D(x-2)^{-2}$$

A third alternative is to use the Chain rule. For this to score there must have been some attempt to divide first to achieve $\mathrm{f}(x)=\frac{4x+1}{x-2}=..+\frac{..}{x-2}$ before applying the chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$

A1 A correct and unsimplified form of the answer.

Accept $\frac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$ from the quotient rule

Accept $\frac{4x-8-4x-1}{(x-2)^2}$ from the quotient rule even if the brackets were missing in line 1

Accept $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}$ or equivalent from the product rule

Accept $9\times -1(x-2)^{-2}$ from the chain rule

A1* Proceeds to achieve the given answer $=\frac{-9}{(x-2)^2}$. Accept $-9(x-2)^{-2}$

All aspects must be correct including the bracketing.

If they differentiated using the product rule the intermediate lines must be seen.

Eg. $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}=\frac{4}{(x-2)}-\frac{4x+1}{(x-2)^2}=\frac{4(x-2)-(4x+1)}{(x-2)^2}=\frac{-9}{(x-2)^2}$

(b)

M1 Sets $\frac{-9}{(x-2)^2}=-1$ and proceeds to $x=....$

The minimum expectation is that they multiply by $(x-2)^2$ and then either, divide by -1 before square rooting or multiply out before solving a 3TQ equation.

A correct answer of $x=5$ would also score this mark following $\frac{-9}{(x-2)^2}=-1$ as long as no incorrect work is seen.

A1 $x=5$

A1 (5, 7) or $x=5, y=7$. Ignore any reference to $x=-1$ (and $y=1$). Do not accept 21/3 for 7

If there is an extra solution, $x>2$, then withhold this final mark.

17.

Question Number	Scheme	Marks
3.(a)	$x = 8\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$	B1*
		(1)
(b)	$\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\frac{dx}{dy} = 8\tan 2\frac{\pi}{8} + 16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\frac{y-\frac{\pi}{8}}{x-\pi} = \frac{1}{8+4\pi}$, accept $y - \frac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \frac{\pi^2}{2}$	A1
		(7)
		(8 marks)

(a)

B1* Either sub $y = \frac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow x = 8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$

Or sub $x = \pi$, $y = \frac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow \pi = 8\times\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi\times 1 = \pi$

This is a proof and therefore an expectation that at least one intermediate line must be seen, including a term in tangent.

Accept as a minimum $y = \frac{\pi}{8} \Rightarrow x = \pi\tan\left(\frac{\pi}{4}\right) = \pi$

Or $\pi = \pi\times\tan\left(\frac{\pi}{4}\right) = \pi$ ✓

This is a given answer however, and as such there can be no errors.

(b)

M1 Applies the product rule to $8y\tan 2y$ achieving $A\tan 2y + By\sec^2(2y)$

A1 One term correct. Either $8\tan 2y$ or $+16y\sec^2(2y)$. There is no requirement for $\frac{dx}{dy} =$

A1 Both lhs and rhs correct. $\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$

It is an intermediate line and the expression does not need to be simplified.

Accept $\frac{dx}{dy} = \tan 2y \times 8 + 8y \times 2\sec^2(2y)$ or $\frac{dy}{dx} = \frac{1}{\tan 2y \times 8 + 8y \times 2\sec^2(2y)}$ or using implicit differentiation $1 = \tan 2y \times 8\frac{dy}{dx} + 8y \times 2\sec^2(2y)\frac{dy}{dx}$

M1 For fully substituting $y = \frac{\pi}{8}$ into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ to find a 'numerical' value

Accept $\frac{dx}{dy} =$ awrt 20.6 or $\frac{dy}{dx} =$ awrt 0.05 as evidence

M1 For a correct attempt at an equation of the tangent at the point $\left(\pi, \frac{\pi}{8}\right)$.

The gradient must be an inverted numerical value of their $\frac{dx}{dy}$

Look for $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{\text{numerical } \frac{dx}{dy}}$,

Watch for negative reciprocals which is M0

If the form $y = mx + c$ is used it must be a full method to find a 'numerical' value to c.

A1 A correct equation of the tangent.

Accept $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{8 + 4\pi}$ or if $y = mx + c$ is used accept $m = \frac{1}{8 + 4\pi}$ and $c = \frac{\pi}{8} - \frac{\pi}{8 + 4\pi}$

Watch for answers like this which are correct $x - \pi = (8 + 4\pi)\left(y - \frac{\pi}{8}\right)$

Accept the decimal answers awrt 2sf $y = 0.049x + 0.24$, awrt 2sf $21y = x + 4.9$, $\frac{y - 0.39}{x - 3.1} = 0.049$

Accept a mixture of decimals and π's for example $20.6\left(y - \frac{\pi}{8}\right) = x - \pi$

A1 Correct answer and solution only. $(8 + 4\pi)y = x + \frac{\pi^2}{2}$

Accept exact alternatives such as $4(2 + \pi)y = x + 0.5\pi^2$ and because the question does not ask for a and b to be simplified in the form $ay = x + b$, accept versions like

$(8 + 4\pi)y = x + \frac{\pi}{8}(8 + 4\pi) - \pi$ and $(8 + 4\pi)y = x + (8 + 4\pi)\left(\frac{\pi}{8} - \frac{\pi}{8 + 4\pi}\right)$

18.

Question Number	Scheme	Marks
8.(a)	$P = \frac{800e^0}{1+3e^0}, = \frac{800}{1+3} = 200$	M1,A1
		(2)
(b)	$250 = \frac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t}) = 800e^{0.1t} \Rightarrow 50e^{0.1t}{=}250, \Rightarrow e^{0.1t} = 5$	M1,A1
	$t = \frac{1}{0.1}\ln(5)$	M1
	$t = 10\ln(5)$	A1
		(4)
(c)	$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow \frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$	M1,A1
	At t=10	
	$\frac{dP}{dt} = \frac{(1+3e)\times 80e - 240e^2}{(1+3e)^2} = \frac{80e}{(1+3e)^2}$	M1,A1
		(4)
(d)	$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow P_{max} = \frac{800}{3} = 266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub $t = 0$ into P **and** use $e^0 = 1$ in at least one of the two cases. Accept $P = \frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P = \frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t} = B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t} = D$

A1 $e^{0.1t} = 5$ or $e^{-0.1t} = 0.2$

M1 Dependent upon gaining $e^{0.1t}=E$, for taking ln's of both sides and proceeding to t=…

Accept $e^{0.1t}=E \Rightarrow 0.1t=\ln E \Rightarrow t=...$It could be implied by $t=\text{awrt } 16.1$

A1 $t=10\ln(5)$

Accept exact equivalents of this as long as *a* and *b* are integers. Eg. $t=5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{d}{dt}e^{0.1t}=ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u=800e^{0.1t}, v=1+3e^{0.1t}, u'=pe^{0.1t}, v'=qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t}-800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{d}{dt}e^{0.1t}=ke^{0.1t}$

For the chain rule look for

$$P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3} \Rightarrow \frac{dP}{dt}=800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{dP}{dt}=\frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t=10$ into their $\frac{dP}{dt}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{dP}{dt}$ was correct

A1 $\frac{dP}{dt}=\frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{dP}{dt}=80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t=10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270=\frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27=e^{0.1t} \Rightarrow 0.1t=\ln(-27)$ which has no answers.

Eg. $270=\frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27=e^{0.1t} \Rightarrow e^{0.1t} / e^{x}$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

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19.

Question Number	Scheme	Marks	
5(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1	**(2)**
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1	
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1	
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1	
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1*	**(4)**
(c)	$\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1	
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1	
			(4)
			(10 marks)

Alt 1 to 5(a)	$x = (\cos 3y)^{-2} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to 5 (a)	$x = \sec 3y \times \sec 3y \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To 5 (c)	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question 5

(a)

M1 Uses the chain rule to get $A\sec 3y \sec 3y \tan 3y = \left(A\sec^2 3y \tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y \sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2 \times 3\sec 3y \sec 3y \tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

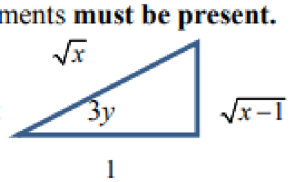
(b)

M1 Uses $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$ to get an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. Follow through on their $\dfrac{\mathrm{d}x}{\mathrm{d}y}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = \left(x-1\right)^{\frac{1}{2}}$

All elements **must be present.**

Accept 

$\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

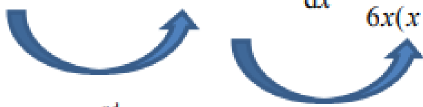
M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 3y \tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$



Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question 5 Continued

Example 1- Scores 0 marks in part (b)

$$\frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 3y \tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6\sec^2 3x \tan 3x} = \frac{1}{6\sec^2 3x\sqrt{\sec^2 3x - 1}} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{\mathrm{d}x}{\mathrm{d}y} = 2\sec^2 3y \tan 3y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sec^2 3y \tan 3y} = \frac{1}{2\sec^2 3y\sqrt{\sec^2 3y - 1}} = \frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu' - uv'}{v^2}$ with $u = 1$ and $v = 6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u' = 0$ and $v' = A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = \frac{0 - [A(x-1)^{\frac{1}{2}} + Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right) = \frac{0 - A(x-1)^{\frac{1}{2}} \pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question 5 Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form $(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\cancel{\frac{d^2y}{dx^2}} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{d^2y}{dx^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) **Using Quotient and Chain rules Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving $u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$$\cancel{\left(\frac{d^2y}{dx^2}\right)} = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$

A1 Correct un simplified expression $\cancel{\frac{d^2y}{dx^2}} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1	Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.
	Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression
	This is dependent upon the 1st M1 being scored.
A1	Correct simplified expression $\frac{d^2y}{dx^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{d^2y}{dx^2}=\frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question 5 Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{dy}{dx}=\frac{1}{6x(x-1)^{\frac{1}{2}}}=\frac{1}{(36x^3-36x^2)^{\frac{1}{2}}}=(36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 3

20.

Question Number	Scheme	Marks
1.	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute **y**=-32 into $y=(2w-3)^5$ **and** proceed to w=….. [Accept slip on sign of y, ie y=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where A is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $\frac{dy}{dx}=10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at P
Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at P. It is dependent upon the previous M mark being awarded.

Award for $'their\ 160' = \dfrac{y-(-32)}{x-their\ '\frac{1}{2}'}$

If they use $y=mx+c$ it must be a full method, using m= 'their 160', their '$\frac{1}{2}$' and -32.

An attempt must be seen to find c=…

A1 cso $y=160x-112$. The question is specific and requires the answer in this form.
You may isw in this question after a correct answer.

21.

Question Number	Scheme	Marks
5.	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$ $= 3x^2 \ln 2x + x^2$	M1A1A1 (3)
	(i)(b) $\frac{dy}{dx} = 3(\boldsymbol{x + \sin 2x})^2 \times (1 + 2\cos 2x)$	**B1**M1A1 (3)
	(ii) $\frac{dx}{dy} = -\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2}$ cso	M1, A1* (5) **(11 marks)**

(i)(a) M1 Applies the product rule vu'+uv' to $x^3 \ln 2x$.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \qquad \text{where } A, B \text{ are constants} \neq 0$$

A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \frac{1}{2x} \times 2$

A1 Cao. $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2(3\ln 2x + 1)$

(i)(b) B1 Sight of $(x + \sin 2x)^2$

M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x + \sin 2x)^2 \times (1 \pm D\cos 2x)$ where C and D are non- zero constants.

Alternatively accept $u = x + \sin 2x$, u'= followed by $Cu^2 \times$ their u'

Do not accept $C(x + \sin 2x)^2 \times 2\cos 2x$ unless you have evidence that this is their u'

Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D\cos 2x$

A1 Cao $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of cot***y*** as -cosec^2***y***. It must be in terms of y

A1 $\frac{dx}{dy} = -\text{cosec}^2 y$ or $1 = -\text{cosec}^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\text{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x.

Note that this may be reached before the previous M, BUT is still recorded as the 4th mark on epen

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$. See main notes

Alternative to 5(ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \{-1 - \cot^2 y\}$$

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \text{cosec}^2 y$ and $\text{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in x

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).

Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

A1 Correct un simplified answer with both lhs and rhs correct.

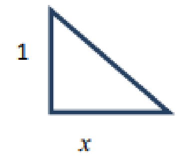
$$\frac{dx}{dy} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \frac{1}{x}$ to find siny

and get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x

$x = \cot y \Rightarrow \tan y = \frac{1}{x}$



$\sin y = \frac{1}{\sqrt{1+x^2}}$

22.

Question Number	Scheme	Marks
7.	(a) $\frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$= \frac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$= \frac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$	M1A1
	$h'(x) = \frac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $h'(x) = 0 \Rightarrow 10 - 2x^2 = 0 \Rightarrow x = ..$	M1
	$\Rightarrow x = \sqrt{5}$	A1
	When $x = \sqrt{5} \Rightarrow h(x) = \frac{\sqrt{5}}{5}$	M1,A1
	Range of h(x) is $0 \le h(x) \le \frac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone 'invisible' brackets for this mark. Accept three separate fractions with the same denominator. Amongst possible options allowed for this method are

$\frac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$ Eg 2An example of a slip (on middle term), 1st term has been adapted

$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)}+\frac{4(x+2)}{(x+2)(x^2+5)}-\frac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator. A factor of $(x+2)$ must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\frac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A, B>0$$

A1 Correct unsimplified answer $h'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $h'(x)=\frac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\frac{2(5-x^2)}{(x^2+5)^2}$ $\frac{-2(x^2-5)}{(x^2+5)^2}$, $\frac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical slips but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$.
Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\frac{\sqrt{5}}{5}$. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0 \le \text{h}(x) \le \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0 \le y \le \frac{\sqrt{5}}{5}$, $0 \le Range \le \frac{\sqrt{5}}{5}$, $\left[0, \frac{\sqrt{5}}{5}\right]$ but not $0 \le x \le \frac{\sqrt{5}}{5}$, $\left(0, \frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow.

Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Any correct answers without working must be sent to team leaders via review

Alternative to (b) using the product rule

M1 Sets $\text{h}(x) = 2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being $2x$ and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1} \times A + 2x \times \pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1} \times 2 + 2x \times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$\text{h}'(x) = \frac{10-2x^2}{(x^2+5)^2} = \frac{2(5-x^2)}{(x^2+5)^2} = \frac{-2(x^2-5)}{(x^2+5)^2} = (10-2x^2)(x^2+5)^{-2}$$

23.

Question Number	Scheme	Marks
8.	(a) (£) 19500	B1 (1)
	(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	
	$17e^{-0.25t} + 2e^{-0.5t} = 9$	
	$(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4\ln(2)$ *oe*	A1 (4)
	(c)	
	$(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1 (4)
		(9 marks)

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct
Accept $17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0$ and $17000x + 2000x^2 - 9000 = 0$ where $x = e^{-0.25t}$

M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t} - 1)(e^{-0.25t} + 9) = 0$
Alternatively let $'x' = e^{0.25t}$ or otherwise and factorise a quadratic equation in x

A1 Correct solution of the quadratic. Either $e^{0.25t} = 2$ or $e^{-0.25t} = \frac{1}{2}$ oe.

A1 Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}, \frac{\ln(2)}{0.25}, -4\ln(\frac{1}{2})$

.(c) M1 Differentiates $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ by the chain rule.
Accept answers of the form $(\frac{dV}{dt}) = \pm Ae^{-0.25t} \pm Be^{-0.5t}$ A, B are constants $\neq 0$

A1 Correct derivative $(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$.
There is no need for it to be simplified so accept

$$(\frac{dV}{dt}) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t} \quad oe$$

M1 Substitute t=8 into their $\frac{dV}{dt}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept t=8 in V

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Watch for candidates who sub t=8 into V first and then differentiate. They sometimes achieve 593. This is M0A0M0A0.**

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24.

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\tan 3x = -\sqrt{3}$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)
	(b) At $x=0$ $\frac{dy}{dx} = 3$	B1
	Equation of normal is $-\frac{1}{3} = \frac{y-0}{x-0}$ or any equivalent $y = -\frac{1}{3}x$	M1A1
		(3) **(9 marks)**

(a) M1 Applies the product rule vu'+uv' to $e^{x\sqrt{3}}\sin 3x$. If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form $\frac{dy}{dx} = Ae^{x\sqrt{3}}\sin 3x + e^{x\sqrt{3}} \times \pm B\cos 3x$

A1 Correct expression for $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$

M1 Sets **their** $\frac{dy}{dx} = 0$, factorises out or divides by $e^{x\sqrt{3}}$ producing an equation in sin3x and cos3x

A1 Achieves either $\tan 3x = -\sqrt{3}$ or $\tan 3x = -\frac{3}{\sqrt{3}}$

M1 Correct order of arctan, followed by ÷3.

Accept $3x = \frac{5\pi}{3} \Rightarrow x = \frac{5\pi}{9}$ or $3x = \frac{-\pi}{3} \Rightarrow x = \frac{-\pi}{9}$ but not $x = \arctan(\frac{-\sqrt{3}}{3})$

A1 CS0 $x = \frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

(b) B1 Sight of **3** for the gradient

M1 A full method for finding an equation of the normal.

Their tangent gradient m must be modified to $-\frac{1}{m}$ and used together with (0, 0).

Eg $-\frac{1}{their\ 'm'}=\frac{y-0}{x-0}$ or equivalent is acceptable

A1 $y=-\frac{1}{3}x$ or any correct equivalent including $-\frac{1}{3}=\frac{y-0}{x-0}$.

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$(\sqrt{12})\sin(3x+\frac{\pi}{3})=0$	A1
	$3x=\frac{2\pi}{3}\Rightarrow x=\frac{2\pi}{9}$	M1A1
		(6)

A1 Achieves either $(\sqrt{12})\sin(3x+\frac{\pi}{3})=0$ or $(\sqrt{12})\cos(3x-\frac{\pi}{6})=0$

M1 Correct order of arcsin or arcos, etc to produce a value of x

Eg accept $3x+\frac{\pi}{3}=0$ or π or $2\pi \Rightarrow x=....$

A1 Cao $x=\frac{2\pi}{9}$ Ignore extra solutions outside the range. Withhold mark for extra inside the range.

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx}=\sqrt{3}e^{x\sqrt{3}}\sin 3x+3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx}=0$ $e^{x\sqrt{3}}(\sqrt{3}\sin 3x+3\cos 3x)=0$	M1
	$\sqrt{3}\sin 3x=-3\cos 3x\Rightarrow \cos^2(3x)=\frac{1}{4}$ or $\sin^2(3x)=\frac{3}{4}$	A1
	$x=\frac{1}{3}\arccos(\pm\sqrt{\frac{1}{4}})$ oe	M1
	$x=\frac{2\pi}{9}$	A1

25.

Question Number	Scheme	Marks
7.	(a)(i) $\frac{\text{d}}{\text{d}x}(\ln(3x))=\frac{3}{3x}$	M1
	$\frac{\text{d}}{\text{d}x}(x^{\frac{1}{2}}\ln(3x))=\ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}}+x^{\frac{1}{2}}\times\frac{3}{3x}$	M1A1
		(3)
	(ii) $\frac{\text{d}y}{\text{d}x}=\frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\frac{\text{d}y}{\text{d}x}=\frac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x=3\tan 2y \quad \Rightarrow \frac{\text{d}x}{\text{d}y}=6\sec^2 2y$	M1A1
	$\Rightarrow \frac{\text{d}y}{\text{d}x}=\frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y=1+\tan^2 2y$ and uses $\tan 2y=\frac{x}{3}$	
	$\Rightarrow \frac{\text{d}y}{\text{d}x}=\frac{1}{6(1+(\frac{x}{3})^2)}=\left(\frac{3}{18+2x^2}\right)$	M1A1
		(5) **(11 marks)**

Note that this is marked B1M1A1 on EPEN

(a)(i) M1 Attempts to differentiate $\ln(3x)$ to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine.

M1 Attempts the product rule for $x^{\frac{1}{2}}\ln(3x)$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$\ln(3x)\times Ax^{-\frac{1}{2}}+x^{\frac{1}{2}}\times\frac{B}{x},\quad A,B>0$

A1 Any correct (un simplified) form of the answer. Remember to isw any incorrect further work

$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x))=\ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}}+x^{\frac{1}{2}}\times\frac{3}{3x}=\left(\frac{\ln(3x)}{2\sqrt{x}}+\frac{1}{\sqrt{x}}\right)=x^{-\frac{1}{2}}\left(\frac{1}{2}\ln 3x+1\right)$

Note that this part does not require the answer to be in its simplest form

(ii) M1 Applies the quotient rule, a version of which appears in the formula booklet. If the formula is quoted it must be correct. There must have been an attempt to differentiate both terms. If the formula is not quoted nor implied from their stating of u, u', v, v' and their subsequent expression, only accept answers of the form

$$\frac{(2x-1)^5\times\pm 10-(1-10x)\times C(2x-1)^4}{(2x-1)^{10\text{ or }7\text{ or }25}}$$

A1 Any un simplified form of the answer. Eg $\frac{dy}{dx}=\frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{((2x-1)^5)^2}$

A1 Cao. It must be simplified as required in the question $\frac{dy}{dx}=\frac{80x}{(2x-1)^6}$

(b) M1 Knows that $3\tan 2y$ differentiates to $C\sec^2 2y$. The lhs can be ignored for this mark. If they write $3\tan 2y$ as $\frac{3\sin 2y}{\cos 2y}$ this mark is awarded for a correct attempt of the quotient rule.

A1 Writes down $\frac{dx}{dy}=6\sec^2 2y$ or implicitly to get $1=6\sec^2 2y\frac{dy}{dx}$

Accept from the quotient rule $\frac{6}{\cos^2 2y}$ or even $\frac{\cos 2y\times 6\cos 2y-3\sin 2y\times -2\sin 2y}{\cos^2 2y}$

M1 An attempt to invert 'their' $\frac{dx}{dy}$ to reach $\frac{dy}{dx}=\text{f}(y)$, or changes the subject of their implicit

. differential to achieve a similar result $\frac{dy}{dx}=\text{f}(y)$

M1 Replaces an expression for $\sec^2 2y$ in their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with x by attempting to use $\sec^2 2y=1+\tan^2 2y$. Alternatively, replaces an expression for y in $\frac{dx}{dy}$ or $\frac{dy}{dx}$ with $\frac{1}{2}\arctan(\frac{x}{3})$

A1 Any correct form of $\frac{dy}{dx}$ in terms of x. $\frac{dy}{dx}=\frac{1}{6(1+(\frac{x}{3})^2)}$ $\frac{dy}{dx}=\frac{3}{18+2x^2}$ *or* $\frac{1}{6\sec^2(\arctan(\frac{x}{3}))}$

Question Number	Scheme	Marks
7.	**(a)(ii) Alt using the product rule** Writes $\frac{1-10x}{(2x-1)^5}$ as $(1-10x)(2x-1)^{-5}$ and applies vu'+uv'. See (a)(i) for rules on how to apply $(2x-1)^{-5}\times -10+(1-10x)\times -5(2x-1)^{-6}\times 2$ Simplifies as main scheme to $80x(2x-1)^{-6}$ or equivalent	 M1A1 A1 (3)
	(b) Alternative using arctan. They must attempt to differentiate to score any marks. Technically this is M1A1M1A2 Rearrange $x=3\tan 2y$ to $y=\frac{1}{2}\arctan(\frac{x}{3})$ **and attempt to differentiate** Differentiates to a form $\frac{A}{1+(\frac{x}{3})^2}$, $=\frac{1}{2}\times\frac{1}{(1+(\frac{x}{3})^2)}\times\frac{1}{3}$ or $\frac{1}{6(1+(\frac{x}{3})^2)}$ oe	 M1A1 M1, A2 (5)

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26.

Question No	Scheme	Marks
1	(a)	
	$\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1
		(4)
	(b)	
	Applying $\frac{vu'-uv'}{v^2}$	
	$\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1 A1
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 MARKS)

(a) M1 Differentiates the $\ln(3x)$ term to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine for this mark.

M1 Applies the product rule to $x^2\ln(3x)$. If the rule is quoted it must be correct.
There must have been some attempt to differentiate both terms.
If the rule is **not quoted (or implied by their working)** only accept answers of the form
$\ln(3x) \times Ax + x^2 \times \frac{B}{x}$ where A and B are non- zero constants

A1 One term correct and simplified, either $2x\ln(3x)$ or x. $\ln 3x^{2x}$ and $\ln(3x)\,2x$ are acceptable forms

A1 Both terms correct and simplified on the same line. $2x\ln(3x) + x$, $\ln(3x) \times 2x + x$, $x(2\ln 3x + 1)$ oe

(b) M1 Applies the quotient rule. A version of this appears in the formula booklet. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the formula is **not quoted (nor implied by their working)** only accept answers of the form

$$\frac{x^3 \times \pm A\cos(4x) - \sin(4x) \times Bx^2}{(x^3)^2 \text{ or } x^6 \text{ or } x^5 \text{ or } x^9} \text{ with } B > 0$$

A1 Correct first term on numerator $x^3 \times 4\cos(4x)$

A1 Correct second term on numerator $-\sin(4x) \times 3x^2$

A1 Correct denominator x^6, the $(x^3)^2$ needs to be simplified

A1 Fully correct simplified expression $\frac{4x\cos(4x) - 3\sin(4x)}{x^4}$, $\frac{\cos(4x)4x - \sin(4x)3}{x^4}$ oe .
Accept $4x^{-3}\cos(4x) - 3x^{-4}\sin(4x)$ oe

Alternative method using the product rule.

M1,A1 **Writes $\frac{sin(4x)}{x^3}$ as** $\sin(4x) \times x^{-3}$ and applies the product rule. They will score both of these marks or neither of them. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form $x^{-3} \times Acos(4x) + \sin(4x) \times \pm Bx^{-4}$

A1 One term correct, either $x^{-3} \times 4cos(4x)$ or $\sin(4x) \times -3x^{-4}$

A1 Both terms correct,Eg. $x^{-3} \times 4cos(4x) + \sin(4x) \times -3x^{-4}$.

A1 Fully correct expression. $4x^{-3}cos(4x) - 3x^{-4}\sin(4x)$ or $4cos(4x)x^{-3} - 3\sin(4x)x^{-4}$ oe
The negative must have been dealt with for the final mark.

27.

Question No	Scheme	Marks
4	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y + \frac{\pi}{12}\right)$	M1,A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y=\frac{\pi}{4}$, $x=2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$ oe	A1 (7 marks)

M1 For differentiation of $2\tan\left(y + \frac{\pi}{12}\right) \rightarrow 2sec^2\left(y + \frac{\pi}{12}\right)$. There is no need to identify this with $\frac{dx}{dy}$

A1 For correctly writing $\frac{dx}{dy} = 2sec^2\left(y + \frac{\pi}{12}\right)$ or $\frac{dy}{dx} = \frac{1}{2sec^2\left(y+\frac{\pi}{12}\right)}$

M1 Substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy}$. Accept if $\frac{dx}{dy}$ is inverted and $y = \frac{\pi}{4}$ substituted into $\frac{dy}{dx}$.

A1 $\frac{dx}{dy} = 8$ or $\frac{dy}{dx} = \frac{1}{8}$ oe

B1 Obtains the value of $x=2\sqrt{3}$ corresponding to $y=\frac{\pi}{4}$. Accept awrt 3.46

M1 This mark requires **all of the necessary elements for** finding **a numerical equation** of the **normal.**
Either Invert their value of $\frac{dx}{dy}$, to find $\frac{dy}{dx}$, then use $m_1 \times m_2 = -1$ to find the numerical gradient of the normal
Or use their numerical value of $-\frac{dx}{dy}$
Having done this then use $\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$
The $2\sqrt{3}$ could appear as awrt 3.46, the $\frac{\pi}{4}$ as awrt 0.79,
This cannot be awarded for finding the equation of a tangent.
Watch for candidates who correctly use $(x - their\ 2\sqrt{3}) = -their\ numerical\ \frac{dy}{dx}\left(y - \frac{\pi}{4}\right)$
If they use 'y=mx+c' it must be a full method to find c.

A1 Any correct form of the answer. It does not need to be simplified and the question does not ask for an exact answer.
$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$, $\frac{y-\frac{\pi}{4}}{x-2\sqrt{3}} = -8$, $y = -8x + \frac{\pi}{4} + 16\sqrt{3}$, $y=-8x+$ (awrt) 28.5

Alternatives using arctan (first 3 marks)

M1 Differentiates $y = \arctan\left(\frac{x}{2}\right) - \frac{\pi}{12}$ to get $\frac{1}{1+(\frac{x}{2})^2} \times constant$. Don't worry about the lhs

A1 Achieves $\frac{dy}{dx} = \frac{1}{1+(\frac{x}{2})^2} \times \frac{1}{2}$

M1 This method mark requires x to be found, which then needs to be substituted into $\frac{dy}{dx}$

The rest of the marks are then the same.

Or implicitly (first 2 marks)

M1 Differentiates implicitly to get $1 = 2\,sec^2\left(y + \frac{\pi}{12}\right) \times \frac{dy}{dx}$

A1 Rearranges to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of y

The rest of the marks are the same

Or by compound angle identities

$$x = 2\tan\left(y + \frac{\pi}{12}\right) = \frac{2tany + 2\tan\left(\frac{\pi}{12}\right)}{1 - tany\; tan\frac{\pi}{12}} \text{ oe}$$

M1 Differentiates using quotient rule-see question 1 in applying this. Additionally the tany **must** have been differentiated to sec^2y. There is no need to assign to $\frac{dx}{dy}$

A1 The correct answer for $\frac{dx}{dy} = \frac{\left(1 - tany\; tan\frac{\pi}{12}\right) \times 2sec^2y - \left(2tany + 2\tan\left(\frac{\pi}{12}\right)\right) \times -sec^2y tan\frac{\pi}{12}}{(1 - tany\; tan\frac{\pi}{12})^2}$

The rest of the marks are as the main scheme

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28.

Question Number	Scheme	Marks
1 (a)	$\frac{1}{(x^2+3x+5)} \times ....\ , = \quad \frac{2x+3}{(x^2+3x+5)}$	M1,A1
		(2)
(b)	Applying $\frac{vu' - uv'}{v^2}$	M1,
	$\frac{x^2 \times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2 \sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0
		(3)
		5 Marks

29.

Question Number	Scheme	Marks
5 (a)	$p=7.5$	B1
		(1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	
		(4)

(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

30.

Question Number	Scheme	Marks
7 (a)	$x^2 - 9 = (x+3)(x-3)$	B1
	$\frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)}$	
	$= \frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \frac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$= \frac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$= \frac{5(x-3)}{(2x+1)(x-3)(x+3)} = \frac{5}{(2x+1)(x+3)}$	A1*
		(5)

(b)	$$f(x) = \frac{5}{2x^2+7x+3}$$	
	$$f'(x) = \frac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$$	M1**M1**A1
	$$f'(-1) = -\frac{15}{4}$$	M1A1
	Uses m_1m_2=-1 to give gradient of normal=$\frac{4}{15}$	M1
	$$\frac{y-(-\frac{5}{2})}{(x--1)} = their\ \frac{4}{15}$$	M1
	$y+\frac{5}{2} = \frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

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31.

Question Number	Scheme		Marks
4.			
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t=0,\ \theta=90 \Rightarrow\}\quad 90 = 20 + Ae^{-k(0)}$	Substitutes $t=0$ and $\theta=90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t=5,\ \theta=55 \Rightarrow\}\quad 55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t=5$ and $\theta=55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm\alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t=10$, $\frac{d\theta}{dt} = -14\ln 2e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ ^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

32.

Question Number	Scheme		Marks
7			
(a)	$y = \dfrac{3 + \sin 2x}{2 + \cos 2x}$		
	Apply quotient rule: $\left\{\begin{array}{ll} u = 3 + \sin 2x & v = 2 + \cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{array}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2 + \cos 2x) - -2\sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3 + \sin \pi}{2 + \cos \pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $(\frac{\pi}{2}, 3)$, $m(\mathbf{T}) = \dfrac{6\sin \pi + 4\cos \pi + 2}{(2 + \cos \pi)^2} = \dfrac{-4+2}{1^2} = -2$	$m(\mathbf{T}) = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) **[8]**

33.

Question Number	Scheme		Marks
8.			
(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG**
			(3)
(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \mathbb{Z}.$		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K \sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			[9]

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34.

Question Number	Scheme	Marks
2.	At P, $y = \underline{3}$	B1
	$\frac{dy}{dx} = \underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$	M1$\underline{\text{A1}}$
	$\frac{dy}{dx} = \frac{18}{(5-3(2))^3}$ $\{= -18\}$	M1
	$m(\mathbf{N}) = \frac{-1}{-18}$ or $\frac{1}{18}$	M1
	$\mathbf{N}$: $y - 3 = \frac{1}{18}(x-2)$	M1
	$\mathbf{N}$: $\underline{x - 18y + 52 = 0}$	A1
		[7]
	1st M1: $\pm k(5-3x)^{-3}$ can be implied. See appendix for application of the quotient rule. 2nd M1: Substituting $x = 2$ into an equation involving their $\frac{dy}{dx}$; 3rd M1: Uses $m(\mathbf{N}) = -\frac{1}{\text{their } m(\mathbf{T})}$. 4th M1: $y - y_1 = m(x-2)$ with 'their NORMAL gradient' or a "changed" tangent gradient and their y_1. Or uses a complete method to express the equation of the tangent in the form $y = mx + c$ with 'their NORMAL ("changed" **numerical**) gradient', their y_1 and $x = 2$. **Note**: To gain the final A1 mark all the previous 6 marks in this question need to be earned. Also there must be a completely correct solution given.	

35.

Question Number	Scheme	Marks
5. (a)	Either $y = 2$ or $(0, 2)$	B1
		(1)
(b)	When $x = 2,\ y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1
	$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$	M1
	Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1
		(3)
(c)	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1A1A1
		(3)
(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	M1
	$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$	M1
	$x = \frac{7}{2}, 1$	A1
	When $x = \frac{7}{2},\ y = 9e^{-\frac{7}{2}}$, when $x = 1,\ y = -e^{-1}$	ddM1A1
		(5) **[12]**

	(b) If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark. (c) M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v') A1: Any one term correct. A1: Both terms correct. (d) 1st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0. 2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$. See rules for solving a three term quadratic equation on page 1 of this Appendix. 3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$. Note that this method mark is dependent on the award of the two previous method marks in this part. Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf. Final A1: Both $\{x = 1\}$, $y = -e^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. **cao** Note that both exact values of y are required.	

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36.

Question Number	Scheme		Marks
Q4 (i)	$y = \dfrac{\ln(x^2+1)}{x}$		
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	$\ln(x^2+1) \to \dfrac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \to \dfrac{2x}{x^2+1}$	A1
	Apply quotient rule: $\left\{ \begin{matrix} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{matrix} \right\}$		
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	Applying $\dfrac{xu' - \ln(x^2+1)v'}{x^2}$ correctly.	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{ \dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1) \right\}$	{Ignore subsequent working.}	
(ii)	$x = \tan y$		
	$\dfrac{dx}{dy} = \sec^2 y$	$\tan y \to \sec^2 y$ or an attempt to differentiate $\dfrac{\sin y}{\cos y}$ using either the quotient rule or product rule.	M1*
		$\dfrac{dx}{dy} = \sec^2 y$	A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ $\left\{= \cos^2 y\right\}$	Finding $\dfrac{dy}{dx}$ by reciprocating $\dfrac{dx}{dy}$.	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$	For writing down or applying the identity $\sec^2 y = 1 + \tan^2 y$, which must be applied/stated completely in y.	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	For the correct proof, leading on from the previous line of working.	A1 AG
			(5)
			[9]

37.

Question Number	Scheme		Marks
Q7 (a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	$\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)} = \underline{\sec x \tan x}$	Convincing proof. Must see both underlined steps.	A1 AG
			(3)
(b)	$y = e^{2x} \sec 3x$		
	$\left\{\begin{array}{ll} u = e^{2x} & v = \sec 3x \\ \dfrac{du}{dx} = 2e^{2x} & \dfrac{dv}{dx} = 3\sec 3x \tan 3x \end{array}\right\}$ Seen or implied	Either $e^{2x} \to 2e^{2x}$ or $\sec 3x \to 3\sec 3x \tan 3x$	M1
		Both $e^{2x} \to 2e^{2x}$ and $\sec 3x \to 3\sec 3x \tan 3x$	A1
	$\dfrac{dy}{dx} = 2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v'	M1
		$2e^{2x}\sec 3x + 3e^{2x}\sec 3x \tan 3x$	A1 isw
			(4)
(c)	Turning point $\Rightarrow \dfrac{dy}{dx} = 0$		
	Hence, $e^{2x}\sec 3x(2 + 3\tan 3x) = 0$	Sets their $\dfrac{dy}{dx} = 0$ and factorises (or cancels) out at least e^{2x} from at least two terms.	M1
	{Note $e^{2x} \neq 0$, $\sec 3x \neq 0$, so $2 + 3\tan 3x = 0$, }		
	giving $\tan 3x = -\frac{2}{3}$	$\tan 3x = \pm k;\ k \neq 0$	M1
	$\Rightarrow 3x = -0.58800 \Rightarrow x = \{a\} = -0.19600...$	Either awrt -0.196^c or awrt -11.2°	A1
	Hence, $y = \{b\} = e^{2(-0.196)}\sec(3 \times -0.196)$		
	$= 0.812093... = 0.812$ (3sf)	0.812	A1 cao
			(4)
			[11]

Part (c): If there are any EXTRA solutions for x (or a) inside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$ or ANY EXTRA solutions for y (or b), (for these values of x) then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $-\frac{\pi}{6} < x < \frac{\pi}{6}$, ie. $-0.524 < x < 0.524$.

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38.

Question Number	Scheme	Notes	Marks
4.	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) **Way 1**	$\left\{\frac{\mathrm{d}x}{\mathrm{d}x} \times\right\}\ \underline{8x - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}}\ \underline{\underline{-4y - 4x\frac{\mathrm{d}y}{\mathrm{d}x}}} + \overline{\overline{2^y \ln 2\frac{\mathrm{d}y}{\mathrm{d}x}}} \underline{= 0}$		M1 A1 M1 B1
	$8(-2) - 3(4)^2\frac{\mathrm{d}y}{\mathrm{d}x} - 4(4) - 4(-2)\frac{\mathrm{d}y}{\mathrm{d}x} + 2^4\ln 2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{\mathrm{d}y}{\mathrm{d}x} - 16 + 8\frac{\mathrm{d}y}{\mathrm{d}x} + 16\ln 2\frac{\mathrm{d}y}{\mathrm{d}x} = 0$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent		A1 **cso**
	NOTE: You can recover work for part (a) in part (b)		**[6]**
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow\ y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N $(\neq m_T)$, either $y - 4 = m_N(x - -2)$ **and** sets $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16},\ \text{so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$		
	$y\ (\text{or } c) = \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ **or** $\frac{13}{2} - \ln 2$ **or** $-\ln 2 + \frac{13}{2}$	A1 **cso isw**
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		**[3]**
			9
(a) **Way 2**	$\left\{\frac{\mathrm{d}x}{\mathrm{d}y} \times\right\}\ \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y} - 3y^2}\ \underline{\underline{-4y\frac{\mathrm{d}x}{\mathrm{d}y} - 4x}} + \overline{\overline{2^y \ln 2}} \underline{= 0}$		M1 A1 M1 B1
	$8(-2)\frac{\mathrm{d}x}{\mathrm{d}y} - 3(4)^2 - 4(4)\frac{\mathrm{d}x}{\mathrm{d}y} - 4(-2) + 2^4\ln 2 = 0$	**dependent on the first M mark**	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent		A1 **cso**
	Note: You must be clear that Way 2 is being applied before you use this scheme		**[6]**

	Question 4 Notes	
4. (a)	**Note**	**For the first four marks**
		Writing down ***from no working***
		• $\frac{dy}{dx} = \frac{4y-8x}{-3y^2-4x+2^y\ln 2}$ or $\frac{8x-4y}{3y^2+4x-2^y\ln 2}$ scores M1A1M1B1
		• $\frac{dy}{dx} = \frac{8x-4y}{-3y^2-4x+2^y\ln 2}$ or $\frac{4y-8x}{3y^2+4x-2^y\ln 2}$ scores M1A0M1B1
		Writing $8x\,dx - 3y^2\,dy - 4y\,dx - 4x\,dy + 2^y\ln 2\,dy = 0$ scores M1A1M1B1

	Question 4 Notes Continued	
4. (a)	**1st M1**	Differentiates implicitly to include ***either*** $\pm 4x\frac{dy}{dx}$ ***or*** $-y^3 \to \pm\lambda y^2\frac{dy}{dx}$ ***or*** $2^y \to \pm\mu 2^y\frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx}=\right)$). λ, μ are constants which can be 1
	1st A1	**Both** $4x^2 - y^3 \to 8x - 3y^2\frac{dy}{dx}$ **and** $=0 \to =0$
	Note	e.g. $8x - 3y^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} + 2^y\ln 2\frac{dy}{dx} \to -3y^2\frac{dy}{dx} - 4x\frac{dy}{dx} + 2^y\ln 2\frac{dy}{dx} = 4y - 8x$ **or** e.g. $-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} \to -48\frac{dy}{dx} + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 32$ will get 1st A1 (implied) as the "$=0$" can be implied by the rearrangement of their equation.
	2nd M1	$-4xy \to -4y - 4x\frac{dy}{dx}$ or $4y - 4x\frac{dy}{dx}$ or $-4y + 4x\frac{dy}{dx}$ or $4y + 4x\frac{dy}{dx}$
	B1	$2^y \to 2^y\ln 2\frac{dy}{dx}$ **or** $2^y \to e^{y\ln 2}\ln 2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0
	3rd dM1	**dependent on the first M mark**
		For substituting $x=-2$ and $y=4$ into an equation involving $\frac{dy}{dx}$
	Note	M1 can be gained by seeing at least one example of substituting $x=-2$ and at least one example of substituting $y=4$ **unless** it is clear that they are instead applying $x=4$ and $y=-2$ Otherwise, you will NEED to check (with your calculator) that $x=-2$, $y=4$ that has been substituted into their equation involving $\frac{dy}{dx}$
	Note	**A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**isw:** You can, however, ignore subsequent working following on from correct solution.

(b)	**Note**	The 2nd M1 mark can be implied by later working. **Eg. Award 1st M1 and 2nd M1** for $\frac{y-4}{2} = \frac{-1}{\text{their } m_T \text{ evaluated at } x=-2 \text{ and } y=4}$
	Note	**A1:** Allow the alternative answer $\{y=\}\ \ln\left(\frac{1}{2}\right) + \frac{13}{2\ln 2}(\ln 2)$ which is in the form $p + q\ln 2$

4. (a) **Way 2**	**1st M1**	Differentiates implicitly to include ***either*** $\pm 4y\frac{dx}{dy}$ ***or*** $4x^2 \to \pm \lambda x\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$). λ is a constant which can be 1
	1st A1	**Both** $4x^2 - y^3 \to 8x\frac{dx}{dy} - 3y^2$ **and** $= 0 \to = 0$
	2nd M1	$-4xy \to -4y\frac{dx}{dy} - 4x$ or $4y\frac{dx}{dy} - 4x$ or $-4y\frac{dx}{dy} + 4x$ or $4y\frac{dx}{dy} + 4x$
	B1	$2^y \to 2^y \ln 2$
	3rd dM1	**dependent on the first M mark** For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dx}{dy}$

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39.

Question Number	Scheme	Notes	Marks
3.	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) **Way 1**	$\left\{\frac{dx}{dx} \times\right\}$ $\left(4xy + 2x^2\frac{dy}{dx}\right) + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$		M1 A1 B1
	$\frac{dy}{dx}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$		dM1
	$\left\{\frac{dy}{dx} = \right\} \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
(b)	At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)}$ $\left\{= \frac{-8}{22 + \pi}\right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ • $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical m_N $(\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\}$ $x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9

(a) **Way 2**	$\left\{\frac{dx}{dy} = \right\} \quad \underline{\underline{\left(4xy\frac{dx}{dy} + 2x^2\right)}} + \underline{2\frac{dx}{dy} + 4 + \pi\sin(\pi y) = 0}$	M1 <u>A1</u> <u><u>B1</u></u>
	$\frac{dx}{dy}(4xy + 2) + 2x^2 + 4 + \pi\sin(\pi y) = 0$	dM1
	$\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$ Correct answer or equivalent	A1 **cso**
		[5]

		Question 3 Notes
3. (a)	**Note**	Writing down ***from no working*** • $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$ scores M1A1B1M1A1 • $\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi\sin(\pi y)}$ scores M1A0B1M1A0
	Note	Few candidates will write $4xy\,dx + 2x^2 dy + 2dx + 4dy + \pi\sin(\pi y)dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or equivalent. This should get full marks.

		Question 3 Notes Continued
3. (a) **Way 1**	**M1**	Differentiates implicitly to include either $2x^2\frac{dy}{dx}$ or $4y \to 4\frac{dy}{dx}$ or $-\cos(\pi y) \to \pm\lambda\sin(\pi y)\frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ is a constant which can be 1.
	1st A1 **Note**	$2x + 4y - \cos(\pi y) = 17 \;\to\; 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = 0$ $4xy + 2x^2\frac{dy}{dx} + 2 + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} \;\to\; 2x^2\frac{dy}{dx} + 4\frac{dy}{dx} + \pi\sin(\pi y)\frac{dy}{dx} = -4xy - 2$ will get 1st A1 (implied) as the "$= 0$" can be implied by the rearrangement of their equation.
	B1	$2x^2y \to 4xy + 2x^2\frac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.
	dM1	**Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. ie. $\frac{dy}{dx}(2x^2 + 4 + \pi\sin(\pi y)) + \;...\; = \;...$
	Note	Writing down an extra $\frac{dy}{dx} = ...$ and then including it in their factorisation is fine for dM1.
	Note	**Final A1 cso:** If the candidate's solution is not completely correct, then do not give this mark.
	Note	**Final A1 isw:** You can, however, ignore subsequent working following on from correct solution.
(a)	**Way 2**	Apply the mark scheme for Way 2 in the same way as Way 1.

(b)	**1st M1**	M1 can be gained by seeing at least one example of substituting $x=3$ and at least one example of substituting $y=\frac{1}{2}$. E.g. "$-4xy$"$\to$"-6" in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x=\frac{1}{2}$, $y=3$.
	3rd M1	***is dependent on the first M1.***
	Note	The 2nd M1 mark can be implied by later working. **Eg. Award 2nd M1 3rd M1** for $\frac{\frac{1}{2}}{3-x}=\frac{-1}{\text{their } m_T}$
	Note	We can accept $\sin\pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2nd M1 mark. But, $\sin\pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of π for the 3rd M1 mark. The 3rd M1 can be accessed for terms containing $\pi\sin\left(\frac{\pi}{2}\right)$.

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40.

Question Number	Scheme		Marks
2.	$x^2-3xy-4y^2+64=0$		
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\times\right\}$ $\underline{2x}-\left(\underline{\underline{3y+3x\frac{dy}{dx}}}\right)\underline{-8y\frac{dy}{dx}}=\underline{0}$		M1<u>A1</u> <u>M1</u>
	$2x-3y+(-3x-8y)\frac{dy}{dx}=0$		dM1
	$\frac{dy}{dx}=\frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ **o.e.**		A1 cso
			[5]
(b)	$\left\{\frac{dy}{dx}=0\Rightarrow\right\}$ $2x-3y=0$		M1
	$y=\frac{2}{3}x$	$x=\frac{3}{2}y$	A1ft
	$x^2-3x\left(\frac{2}{3}x\right)-4\left(\frac{2}{3}x\right)^2+64=0$	$\left(\frac{3}{2}y\right)^2-3\left(\frac{3}{2}y\right)y-4y^2+64=0$	dM1
	$x^2-2x^2-\frac{16}{9}x^2+64=0\Rightarrow-\frac{25}{9}x^2+64=0$	$\frac{9}{4}y^2-\frac{9}{2}y^2-4y^2+64=0\Rightarrow-\frac{25}{4}y^2+64=0$	
	$\left\{\Rightarrow x^2=\frac{576}{25}\Rightarrow\right\}$ $x=\frac{24}{5}$ or $-\frac{24}{5}$	$\left\{\Rightarrow y^2=\frac{256}{25}\Rightarrow\right\}$ $y=\frac{16}{5}$ or $-\frac{16}{5}$	A1 cso
	When $x=\pm\frac{24}{5}$, $y=\frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$	When $y=\pm\frac{16}{5}$, $x=\frac{3}{2}\left(\frac{16}{5}\right)$ and $-\frac{3}{2}\left(\frac{16}{5}\right)$	
	$\left(\frac{24}{5},\frac{16}{5}\right)$ **and** $\left(-\frac{24}{5},-\frac{16}{5}\right)$ **or** $x=\frac{24}{5}$, $y=\frac{16}{5}$ **and** $x=-\frac{24}{5}$, $y=-\frac{16}{5}$ **cso**		ddM1 A1
			[6] **11**

(a)	**Alternative method for part (a)**	
	$\left\{\frac{dx}{dy}\times\right\}$ $\underline{2x\frac{dx}{dy}} - \left(\underline{\underline{3y\frac{dx}{dy} + 3x}}\right) \underline{- 8y} = \underline{0}$	M1A1 M1
	$(2x-3y)\frac{dx}{dy} - 3x - 8y = 0$	dM1
	$\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ **o.e.**	A1 **cso**
		[5]

	Question 2 Notes	
2. (a) **General**	**Note**	Writing down $\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ from **no working** is full marks
	Note	Writing down $\frac{dy}{dx} = \frac{2x-3y}{-3x-8y}$ or $\frac{3y-2x}{3x+8y}$ from **no working** is M1A0B1M1A0
	Note	Few candidates will write $2x\mathrm{d}x - 3y\mathrm{d}x - 3x\mathrm{d}y - 8y\mathrm{d}y = 0$ leading to $\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$, o.e. This should get full marks.

2. (a)	**M1**	Differentiates implicitly to include either $\pm 3x\frac{dy}{dx}$ or $-4y^2 \to \pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	**Both** $x^2 \to \underline{2x}$ **and** $\ldots - 4y^2 + 64 = 0 \to \underline{-8y\frac{dy}{dx} = 0}$
	Note	If an extra term appears then award A0.
	M1	$-3xy \to -3x\frac{dy}{dx} - 3y$ or $-3x\frac{dy}{dx} + 3y$ or $3x\frac{dy}{dx} - 3y$ or $3x\frac{dy}{dx} + 3y$
	Note	$2x - 3y - 3x\frac{dy}{dx} - 8y\frac{dy}{dx} \to 2x - 3y = 3x\frac{dy}{dx} + 8y\frac{dy}{dx}$ will get 1st A1 (implied) as the "$= 0$" can be implied by the rearrangement of their equation.
	dM1	**dependent on the FIRST method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. i.e. $\ldots + (-3x - 8y)\frac{dy}{dx} = \ldots$ or $\ldots = (3x + 8y)\frac{dy}{dx}$. (Allow combining in 1 variable).
	A1	$\frac{2x-3y}{3x+8y}$ or $\frac{3y-2x}{-3x-8y}$ or equivalent.
	Note	**cso** If the candidate's solution is not completely correct, then do not give this mark.
	Note	You cannot recover work for part (a) in part (b).

2. (b)	**M1**	Sets their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or the denominator of their $\frac{\mathrm{d}x}{\mathrm{d}y}$ equal to zero) o.e.
	Note	1st M1 can also be gained by setting $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero in their "$2x-3y-3x\frac{\mathrm{d}y}{\mathrm{d}x}-8y\frac{\mathrm{d}y}{\mathrm{d}x}=0$"
	Note	**If their numerator involves one variable only then only the 1st M1 mark is possible in part (b).**
	Note	**If their numerator is a constant then no marks are available in part (b)**
	Note	**If their numerator is in the form** $\pm ax^2 \pm by = 0$ or $\pm ax \pm by^2 = 0$ **then the first 3 marks are possible in part (b).**
	Note	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-3y}{3x+8y}=0$ is not sufficient for M1.
	A1ft	Either • Sets $2x-3y$ to zero and obtains either $y=\frac{2}{3}x$ or $x=\frac{3}{2}y$ • the follow through result of making either y or x the subject from setting their numerator of their $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero
	dM1	**dependent on the first method mark being awarded.** Substitutes ***either*** their $y=\frac{2}{3}x$ ***or*** their $x=\frac{3}{2}y$ into the original equation to give an equation in one variable only.
	A1	Obtains either $x=\frac{24}{5}$ or $-\frac{24}{5}$ or $y=\frac{16}{5}$ or $-\frac{16}{5}$, (or equivalent) ***by correct solution only***. i.e. You can allow for example $x=\frac{48}{10}$ or 4.8, etc.
	Note	$x=\sqrt{\frac{576}{25}}$ (not simplified) or $y=\sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.

2. (b) ctd	**ddM1**	**dependent on both previous method marks being awarded in this part.** **<u>Method 1</u>** **Either:** • substitutes their x into their $y=\frac{2}{3}x$ **or** substitutes their y into their $x=\frac{3}{2}y$, **or** • substitutes ***the other of*** their $y=\frac{2}{3}x$ ***or*** their $x=\frac{3}{2}y$ into the original equation, **and achieves either:** • exactly two sets of two coordinates **or** • exactly two distinct values for x and exactly two distinct values for y. **<u>Method 2</u>** **Either:** • substitutes their first x-value, x_1 into $x^2-3xy-4y^2+64=0$ to obtain one y-value, y_1 and substitutes their second x-value, x_2 into $x^2-3xy-4y^2+64=0$ to obtain 1 y-value y_2 **or** • substitutes their first y-value, y_1 into $x^2-3xy-4y^2+64=0$ to obtain one x-value x_1 and substitutes their second y-value, y_2 into $x^2-3xy-4y^2+64=0$ to obtain one x-value x_2.
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.

	A1	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$, only **by cso.** Note that decimal equivalents are fine.
	Note	Also allow $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$ all seen in their working to part (b).
	Note	Allow $x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ for 3rd A1.
	Note	$x = \pm\frac{24}{5}$, $y = \pm\frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ **and** $\left(-\frac{16}{5}, -\frac{24}{5}\right)$ (eg. coordinates stated the wrong way round) is 3rd A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ **and** $(-4.8, -3.2)$.
	Note	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.
	Note	No credit in this part can be gained by only setting the denominator to zero.

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41.

Question Number	Scheme	Marks
1.	$x^3 + 2xy - x - y^3 - 20 = 0$	
(a)	$\left\{\frac{\cancel{dx}}{\cancel{dx}} \times\right\}$ $\underline{3x^2} + \left(\underline{\underline{2y + 2x\frac{dy}{dx}}}\right) \underline{- 1 - 3y^2\frac{dy}{dx} = 0}$	M1 <u>A1</u> <u>B1</u>
	$3x^2 + 2y - 1 + (2x - 3y^2)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]
(b)	At $P(3, -2)$, $m(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}$; $= \frac{22}{6}$ or $\frac{11}{3}$	
	and either T: $y - -2 = \frac{"11"}{3}(x - 3)$ **see notes**	M1
	or $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$	
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 **cso**
		[2] 7

(a)	**Alternative method for part (a)**		
	$\left\{\frac{\cancel{dx}}{\cancel{dx}} \times\right\}$ $3x^2\frac{dx}{dy} + \left(2y\frac{dx}{dy} + 2x\right) - \frac{dx}{dy} - 3y^2 = 0$		M1 A1 B1
	$2x - 3y^2 + (3x^2 + 2y - 1)\frac{dx}{dy} = 0$		dM1
	$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$		A1 cso
			[5]
	Question 1 Notes		
(a) General	Note	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ from **no working** is full marks.	
	Note	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from **no working is** M1A0B0M1A0	
	Note	Few candidates will write $3x^2 + 2y + 2x\,dy - 1 - 3y^2dy = 0$ leading to $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$, o.e. This should get full marks.	
1. (a)	**M1**	Differentiates implicitly to include either $2x\frac{dy}{dx}$ or $-y^3 \to \pm k y^2 \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).	
	A1	$x^3 \to 3x^2$ **and** $-x - y^3 - 20 = 0 \to -1 - 3y^2\frac{dy}{dx} = 0$	
	B1	$2xy \to 2y + 2x\frac{dy}{dx}$	
	Note	If an extra term appears then award 1st A0.	

1. (a) ctd	**Note**	$3x^2 + 2y + 2x\frac{dy}{dx} - 1 - 3y^2\frac{dy}{dx} \to 3x^2 + 2y - 1 = 3y^2\frac{dy}{dx} - 2x\frac{dy}{dx}$ will get 1st A1 (implied) as the "$= 0$" can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dy}{dx}$ as long as there are ***at least two terms*** in $\frac{dy}{dx}$. ie. $... + (2x - 3y^2)\frac{dy}{dx} = ...$
	Note	Placing an extra $\frac{dy}{dx}$ at the beginning and then including it in their factorisation is fine for dM1.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ **cso:** If the candidate's solution is not completely correct, then do not give this mark. **isw:** You can, however, ignore subsequent working following on from correct solution.
1. (b)	**M1**	**Some** attempt to substitute **both** $x = 3$ **and** $y = -2$ into their $\frac{dy}{dx}$ which contains both x and y to find m_T **and** • **either** applies $y - -2 = (\text{their } m_T)(x - 3)$, where m_T is a numerical value. • **or** finds c by solving $(-2) = (\text{their } m_T)(3) + c$, where m_T is a numerical value.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ is M0).
	A1	Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.
	cso	A correct solution is required from a correct $\frac{dy}{dx}$.
	isw	You can ignore subsequent working following a correct solution.

	Alternative method for part (a): Differentiating with respect to y	
1. (a)	**M1**	Differentiates implicitly to include either $2y\frac{dx}{dy}$ or $x^3 \to \pm kx^2\frac{dx}{dy}$ or $-x \to -\frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$).
	A1	$x^3 \to 3x^2\frac{dx}{dy}$ **and** $-x - y^3 - 20 = 0 \to -\frac{dx}{dy} - 3y^2 = 0$
	B1	$2xy \to 2y\frac{dx}{dy} + 2x$
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{dx}{dy}$ as long as there are ***at least two terms*** in $\frac{dx}{dy}$.
	A1	For $\frac{1-2y-3x^2}{2x-3y^2}$ or equivalent. Eg: $\frac{3x^2+2y-1}{3y^2-2x}$ **cso:** If the candidate's solution is not completely correct, then do not give this mark.

42.

Question Number	Scheme		Marks
4.	$\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,		
	$\frac{dV}{dh} = 8\pi h + 16\pi$	$\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$ $8\pi h + 16\pi$	M1 A1
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}$ $(8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}$ $\frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$	$\left(\text{Candidate's } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div$ Candidate's $\frac{dV}{dh}$	M1 oe
	When $h = 6$, $\left\{\frac{dh}{dt} = \right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi$ $\left\{ = \frac{80\pi}{64\pi} \right\}$	**dependent on the previous M1 see notes**	dM1
	$\frac{dh}{dt} = \underline{1.25}$ (cms^{-1})	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe
			[5] **5**
	Alternative Method for the first M1A1 Product rule: $\left\{\begin{matrix} u = 4\pi h & v = h+4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{matrix}\right\}$		
	$\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$	$\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$ $4\pi(h+4) + 4\pi h$	M1 A1

		Question 4 Notes
	M1	An expression of the form $\pm\alpha h \pm \beta$, $\alpha \neq 0$, $\beta \neq 0$. **Can be simplified or un-simplified.**
	A1	Correct simplified or un-simplified differentiation of V. eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
	Note	Some candidates will use the product rule to differentiate V with respect to h. **(See Alt Method 1).**
	Note	$\frac{\mathrm{d}V}{\mathrm{d}h}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.
	M1	$\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$
	Note	Also allow 2nd M1 for $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$
	Note	Give 2nd M0 for $\left(\text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}\right) \times \frac{\mathrm{d}h}{\mathrm{d}t} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{\mathrm{d}V}{\mathrm{d}h}$
	dM1	**which is dependent on the previous M1 mark.** Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{\mathrm{d}V}{\mathrm{d}h}$ and 80π (or 80)
	A1	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
	Note	$\frac{80\pi}{64\pi}$ as a final answer is A0.
	Note	Substituting $h = 6$ into a correct $\frac{\mathrm{d}V}{\mathrm{d}h}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)

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43.

Question Number	Scheme	Marks
7.	$x^2 + 4xy + y^2 + 27 = 0$	
(a)	$\left\{\frac{dx}{dx}\times\right\}$ $\quad \underline{2x} + \left(\underline{\underline{4y + 4x\frac{dy}{dx}}}\right) \underline{+ 2y\frac{dy}{dx}} = \underline{0}$	M1 A1 B1
	$2x + 4y + (4x + 2y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y} \quad \left\{= \frac{-x - 2y}{2x + y}\right\}$	A1 **cso oe**
		[5]
(b)	$4x + 2y = 0$	M1
	$y = -2x$; $x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$; $\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$; $-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$; $y^2 = 36$	dM1*
	$x = -3$; $y = 6$	A1
	When $x = -3$, $y = -2(-3)$; When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$; $x = -3$	A1 **cso**
		[7]
		12

	Notes for Question 7
(a)	**M1**: Differentiates implicitly to include either $4x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1: $(x^2) \to (\underline{2x})$ and $\left(... + y^2 + 27 = 0 \to \underline{+ 2y\frac{dy}{dx} = 0}\right)$.
	Note: If an extra term appears then award A0.
	Note: The "= 0" can be implied by rearrangement of their equation.
	i.e.: $2x + 4y + 4x\frac{dy}{dx} + 2y\frac{dy}{dx}$ leading to $4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$ will get A1 (implied).
	B1: $4y + 4x\frac{dy}{dx}$ or $4\left(y + x\frac{dy}{dx}\right)$ or equivalent
	dM1: An attempt to factorise out $\frac{dy}{dx}$ as long as there are at least two terms in $\frac{dy}{dx}$.
	ie. $... + (4x + 2y)\frac{dy}{dx} = ...$ or $... + 2(2x + y)\frac{dy}{dx} = ...$
	Note: This mark is dependent on the previous method mark being awarded.
	A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x - 2y}{2x + y}$
	cso: If the candidate's solution is not completely correct, then do not give this mark.

Notes for Question 7 Continued	
(b)	**M1:** Sets the denominator of their $\frac{dy}{dx}$ equal to zero (or the numerator of their $\frac{dx}{dy}$ equal to zero) oe. **A1:** Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only). The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$. **M1*:** Substitutes $y = \pm\lambda x$ or or $x = \pm\mu y$ or $y = \pm\lambda x \pm a$ or $x = \pm\mu y \pm b$ $(\lambda \neq 0, \mu \neq 0)$ into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable. **dM1*:** leading to at least either $x^2 = A$, $A > 0$ or $y^2 = B$, $B > 0$ **Note:** This mark is dependent on the previous method mark (M1*) being awarded. **A1:** For $x = -3$ (ignore $x = 3$) or if y was found first, $y = 6$ (ignore $y = -6$) (correct solution only). **ddM1*** Substitutes their value of x into $y = \pm\lambda x$ to give $y =$ value or substitutes their value of x into $x^2 + 4xy + y^2 + 27 = 0$ to give $y =$ value. **Alternatively,** substitutes their value of y into $x = \pm\mu y$ to give $x =$ value or substitutes their value of y into $x^2 + 4xy + y^2 + 27 = 0$ to give $x =$ value **Note:** This mark is dependent on the two previous method marks (M1* and dM1*) being awarded. **A1:** $(-3, 6)$ **cso.** **Note:** If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. **DO NOT APPLY ISW ON THIS OCCASION.** **Note:** $x = -3$ followed later in working by $y = 6$ is fine for A1. **Note:** $y = 6$ followed later in working by $x = -3$ is fine for A1. **Note:** $x = -3, 3$ followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$
	Note: Candidates who set the numerator of $\frac{dy}{dx}$ equal to 0 (or the denominator of their $\frac{dx}{dy}$ equal to zero) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ { or even $(6, -3)$ }. **Note:** Candidates who set ***the numerator*** or ***the denominator*** of $\frac{dy}{dx}$ equal to $\pm k$ (usually $k = 1$) can ***only achieve a maximum of 3 marks*** in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0. **Special Case:** It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{dy}{dx}$) to gain all 7 marks in part (b). Eg: An incorrect part (a) answer of $\frac{dy}{dx} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.

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44.

Question Number	Scheme		Marks	
2.	(a) $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ ✱	cso	B1	**(1)**
	(b) $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$		M1	
	At $x = 8$ $\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025 \quad \left(\text{cm s}^{-1}\right)$	2.5×10^{-4}	A1	**(2)**
	(c) $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$		B1	
	$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x\left(\frac{0.048}{3x^2}\right)$		M1	
	At $x = 8$ $\frac{dS}{dt} = 0.024 \quad \left(\text{cm}^2\ \text{s}^{-1}\right)$		A1	**(3)**
				[6]

45.

Question Number	Scheme		Marks	
5.	(a) Differentiating implicitly to obtain $\pm ay^2 \frac{dy}{dx}$ and/or $\pm bx^2 \frac{dy}{dx}$		M1	
	$48y^2 \frac{dy}{dx} + \ \dots\ - 54 \ \dots$		A1	
	$9x^2 y \to 9x^2 \frac{dy}{dx} + 18xy$	or equivalent	B1	
	$\left(48y^2 + 9x^2\right)\frac{dy}{dx} + 18xy - 54 = 0$		M1	
	$\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2} \quad \left(= \frac{18 - 6xy}{16y^2 + 3x^2}\right)$		A1	**(5)**

	(b) $18-6xy=0$	M1
	Using $x=\frac{3}{y}$ or $y=\frac{3}{x}$	
	$16y^3+9\left(\frac{3}{y}\right)^2 y-54\left(\frac{3}{y}\right)=0$ or $16\left(\frac{3}{x}\right)^3+9x^2\left(\frac{3}{x}\right)-54x=0$	M1
	Leading to	
	$16y^4+81-162=0$ or $16+x^4-2x^4=0$	M1
	$y^4=\frac{81}{16}$ or $x^4=16$	
	$y=\frac{3}{2},-\frac{3}{2}$ or $x=2,-2$	A1 A1
	Substituting either of their values into $xy=3$ to obtain a value of the other variable.	M1
	$\left(2,\frac{3}{2}\right),\left(-2,-\frac{3}{2}\right)$ both	A1 (7)
		[12]

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46.

Question Number	Scheme	Marks
1. (a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\times\right\}$ $\underline{2+6y\frac{dy}{dx}}+\underline{\underline{\left(6xy+3x^2\frac{dy}{dx}\right)}}=\underline{8x}$	M1 A1 B1
	$\left\{\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $m(\mathbf{T})=\frac{dy}{dx}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 **cso**
		[5]
(b)	So, $m(\mathbf{N})=\frac{-1}{-\frac{4}{9}}\left\{=\frac{9}{4}\right\}$	M1
	N: $y-1=\frac{9}{4}(x+1)$	M1
	N: $9x-4y+13=0$	A1
		[3] **8**

(a)

M1: Differentiates implicitly to include either $\pm ky\frac{\mathrm{d}y}{\mathrm{d}x}$ or $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$. (Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$).

A1: $(2x+3y^2) \to \left(\underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}\right)$ and $(4x^2 \to \underline{8x})$. **Note:** If an extra "sixth" term appears then award A0.

B1: $6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.

dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly.

If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.

Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$

Note that this mark is dependent on the previous method mark being awarded.

A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44

If the candidate's solution is not completely correct, then do not give this mark.

(b)

M1: Applies $\mathrm{m}(\mathbf{N}) = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$.

M1: Uses $y - 1 = (m_N)(x - -1)$ or finds c using $x=-1$ and $y=1$ and uses $y = (\mathrm{m}_N)x + "c"$,

Where $m_N = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = -\text{their } \mathrm{m}(\mathbf{T})$.

A1: $9x - 4y + 13 = 0$ or $-9x + 4y - 13 = 0$ or $4y - 9x - 13 = 0$ or $18x - 8y + 26 = 0$ etc.

Must be "$= 0$". So do not allow $9x + 13 = 4y$ etc.

Note: $m_N = -\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.

__Alternative method for part (a): Differentiating with respect to y__

$\left\{ \cancel{\frac{\mathrm{d}x}{\mathrm{d}y}} \times \right\}$ $\underline{2\frac{\mathrm{d}x}{\mathrm{d}y} + 6y} + \left(\underline{\underline{6xy\frac{\mathrm{d}x}{\mathrm{d}y} + 3x^2}}\right) = \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}$

M1: Differentiates implicitly to include either $2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $6xy\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} = \right)$).

A1: $(2x+3y^2) \to \left(\underline{2\frac{\mathrm{d}x}{\mathrm{d}y} + 6y}\right)$ and $\left(4x^2 \to \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}\right)$. **Note:** If an extra "sixth" term appears then award A0.

B1: $6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.

dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly.

If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.

Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$

Note that this mark is dependent on the previous method mark being awarded.

A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44

If the candidate's solution is not completely correct, then do not give this mark.

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47.

Question Number	Scheme		Marks	
3.	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent	M1 A1	
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$	M1 A1	(4)
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a)	M1	
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031	A1	(2)
				[6]

48.

Question Number	Scheme		Marks	
5.	$\frac{1}{y}\frac{dy}{dx} = \ldots$		B1	
	$\ldots = 2\ln x + 2x\left(\frac{1}{x}\right)$		M1 A1	
	At $x = 2$, $\ln y = 2(2)\ln 2$		M1	
	leading to $y = 16$	Accept $y = e^{4\ln 2}$	A1	
	At $(2, 16)$ $\frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$		M1	
	$\frac{dy}{dx} = 16(2 + 2\ln 2)$		A1	(7)
				[7]

	Alternative		
	$y = e^{2x\ln x}$	B1	
	$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1	
	$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$	M1 A1	
	At $x = 2$, $\frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$	M1	
	$= 16(2 + 2\ln 2)$	A1	(7)

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49.

2.	$\frac{dI}{dt} = -16\ln(0.5)0.5^t$	M1 A1	
	At $t = 3$ $\frac{dI}{dt} = -16\ln(0.5)0.5^3$	M1	
	$= -2\ln 0.5 = \ln 4$	M1 A1	
			[5]

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50.

Question Number	Scheme	Marks
3.	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2^x\right) = \ln 2.2^x$	B1
	$\ln 2.2^x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = 4 + 6\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 **(7)**
		[7]

51.

Question Number	Scheme	Marks
8.	(a) $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$	M1 A1
	$V = 9\pi h \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}t} = 9\pi\frac{\mathrm{d}h}{\mathrm{d}t}$	B1
	$9\pi\frac{\mathrm{d}h}{\mathrm{d}t} = 0.48\pi - 0.6\pi h$	M1
	Leading to $75\frac{\mathrm{d}h}{\mathrm{d}t} = 4 - 5h$ * cso	A1 **(5)**
	(b) $\int\frac{75}{4-5h}\mathrm{d}h = \int 1\,\mathrm{d}t$ separating variables	M1
	$-15\ln(4-5h) = t \ (+C)$	M1 A1
	$-15\ln(4-5h) = t + C$	
	When $t = 0$, $h = 0.2$	
	$-15\ln 3 = C$	M1
	$t = 15\ln 3 - 15\ln(4-5h)$	
	When $h = 0.5$	
	$t = 15\ln 3 - 15\ln 1.5 = 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	M1 A1

	Alternative for last 3 marks	
	$t = \left[-15\ln(4-5h)\right]_{0.2}^{0.5}$	
	$= -15\ln 1.5 + 15\ln 3$	M1 M1
	$= 15\ln\left(\frac{3}{1.5}\right) = 15\ln 2$ awrt 10.4	A1 **(6)**

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52.

Question Number	Scheme	Marks
Q3	(a) $-2\sin 2x - 3\sin 3y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}$, $\frac{-2\sin 2x}{3\sin 3y}$	A1 (3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$	M1
	$\cos 3y = \frac{1}{2}$	A1
	$3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349	A1 (3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$	M1
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$	M1
	Leading to $6x + 9y - 2\pi = 0$	A1 (3) **[9]**

53.

Question Number	Scheme	Marks
Q6	$\frac{dA}{dt} = 1.5$	B1
	$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$	B1
	When $A = 2$	
	$2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}}$ $(= 0.797\,884\ \ldots)$	M1
	$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	
	$1.5 = 2\pi r \frac{dr}{dt}$	M1
	$\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$ awrt 0.299	A1
		[5]