Numerical Methods- MS

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$	M1	2.1
	$f(3) = (2\ln(5) - x) = (+)0.22$ and $f(4) = (2\ln(4) - 4) = -1.23$ Change of sign and function continuous in interval $[3,4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x) - x)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

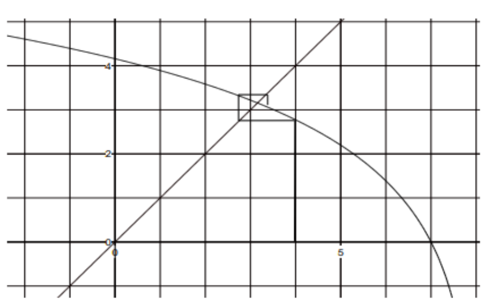
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2ln8 = 3.21 > 3$, $2\ln 4 = 2.77 < 4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

2.

Question Number	Scheme	Marks
5. (a)	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{dy}{dx}\right\|_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}(x-(-2))$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\frac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\frac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
(c)	Substitutes $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929$.	A1
		(2)
		(10 marks)

(a)

B1 $y=3$ at point P. This may be seen embedded within their equation which may be a tangent

M1 Differentiates $\ln(2x+5) \to \frac{A}{2x+5}$ or equivalent. You may see $\ln(2x+5)^2 \to \frac{A(2x+5)}{(2x+5)^2}$

A1 $\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$ oe. It need not be simplified.

M1 For using a correct method of finding the equation of the normal using their numerical value of $-\left.\frac{dx}{dy}\right|_{x=-2}$ as the gradient. Allow for $(y-'3')=-\left.\frac{dx}{dy}\right|_{x=-2}(x--2)$, oe.

At least one bracket must be correct for their $(-2,3)$

If the form $y=mx+c$ is used it is scored for proceeding as far as $c=..$

A1 $\pm k(5y+2x=11)$ It must be in the form $ax+by=c$ as stated in the question

Score this mark once it is seen. Do not withhold it if they proceed to another form, $y=mx+c$ for example

If a candidate uses a graphical calculator to find the gradient they can score a maximum of B1 M0 A0 M1 A1

(b)

M1 For combining 'their' **linear** $5y+2x=11$ with $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in just *x*, condoning slips on the rearrangement of their $5y+2x=11$. Eg $2\ln(2x+5)-\frac{3x}{2}=\frac{11\pm 2x}{5}$ is OK

dM1 Collects the two terms in *x* and proceeds to $ax=b\ln(2x+5)+c$ Allow numerical slips

A1* This is a given answer. All aspects must be correct including bracketing

(c)

M1 Score for substituting $x_1=2\Rightarrow x_2=\frac{20}{11}\ln(2\times 2+5)-2$ or exact equivalent

This may implied by $x_2=1.99...$

A1 Both values correct. Allow awrt $x_2=1.9950$ and $x_3=1.9929$ but condone $x_2=1.995$

Ignore subscripts. Mark on the first and second values given.

June 2016 Mathematics Advanced Paper 1: Pure Mathematics 3

3.

Question	Scheme	Marks
4.(a)	(i) 21	B1
	(ii) $4e^{2x}-25=0\Rightarrow e^{2x}=\frac{25}{4}\Rightarrow 2x=\ln\left(\frac{25}{4}\right)\Rightarrow x=\frac{1}{2}\ln\left(\frac{25}{4}\right),\Rightarrow x=\ln\left(\frac{5}{2}\right)$	M1A1, A1
	(iii) 25	B1
		(5)
(b)	$4e^{2x}-25=2x+43\Rightarrow e^{2x}=\frac{1}{2}x+17$	M1
	$\Rightarrow 2x=\ln\left(\frac{1}{2}x+17\right)\Rightarrow x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$	A1*
		(2)
(c)	$x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)=awrt\ 1.44$	M1
	awrt $x_1=1.4368, x_2=1.4373$	A1
		(2)
(d)	Defines a suitable interval 1.4365 and 1.4375	M1
	...and substitutes into a suitable function Eg $4e^{2x}-2x-68$, obtains correct values with both a reason and conclusion	A1
		(2)
		(11 marks)

In part (a) accept points marked on the graph. If they appear on the graph and in the text, the text takes precedence. If they don't mark (a) as (i) (ii) and (iii) mark in the order given. If you feel unsure then please use the review system and your team leader will advise.

(a) (i)

B1 Sight of 21. Accept $(0,21)$

Do not accept just $|4-25|$ or $(21,0)$

(a) (ii)

M1 Sets $4e^{2x}-25=0$ and proceeds via $e^{2x}=\frac{25}{4}$ or $e^{x}=\frac{5}{2}$ to $x=..$

Alternatively sets $4e^{2x}-25=0$ and proceeds via $(2e^{x}-5)(2e^{x}+5)=0$ to $e^{x}=..$

A1 $\frac{1}{2}\ln\left(\frac{25}{4}\right)$ or awrt 0.92

A1 cao $\ln\left(\frac{5}{2}\right)$ or $\ln 5-\ln 2$. Accept $\left(\ln\left(\frac{5}{2}\right),0\right)$

(a) (iii)

B1 $k=25$ Accept also 25 or $y=25$

Do not accept just $|-25|$ or $x=25$ or $y=\pm 25$

(b)

M1 Sets $4e^{2x}-25=2x+43$ and makes e^{2x} the subject. Look for $e^{2x}=\frac{1}{4}(2x+43+25)$ condoning sign slips.Condone $|4e^{2x}-25|=2x+43$ and makes $|e^{2x}|$ the subject. Condone for both marks a solution with $x=a/\alpha$

An acceptable alternative is to proceed to $2e^{2x}=x+34\Rightarrow \ln 2+2x=\ln(x+34)$ using ln laws

A1* Proceeds correctly without errors to the correct solution. This is a given answer and the bracketing must be correct throughout. The solution must have come from $4e^{2x}-25=2x+43$ with the modulus having been taken correctly.

Allow $e^{2x}=\frac{1}{4}(2x+43+25)$ going to $x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ without explanation

Allow $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ appearing as $\frac{1}{2}\log_e\left(\frac{1}{2}x+17\right)$ but not as $\frac{1}{2}\log\left(\frac{1}{2}x+17\right)$

....

If a candidate attempts the solution backwards they must proceed from

$x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)\Rightarrow e^{2x}=\frac{1}{2}x+17\Rightarrow 4e^{2x}-25=2x+43$ for the M1

For the A1 it must be tied up with a minimal statement that this is $g(x)=2x+43$

....

(c)

M1 Subs 1.4 into the iterative formula in an attempt to find x_1

Score for $x_1 = \frac{1}{2}\ln\left(\frac{1}{2}\times 1.4 + 17\right)$ $x_1 = \frac{1}{2}\ln(17.7)$ or awrt 1.44

A1 awrt $x_1 = 1.4368$, $x_2 = 1.4373$ Subscripts are not important, mark in the order given please.

(d)

M1 For a suitable interval. Accept 1.4365 and 1.4375 (or any two values of a smaller range spanning the root=1.4373) Continued iteration is M0

A1 Substitutes both values into **a suitable function**, which must be defined or implied by their working calculates both values correctly to 1 sig fig (rounded or truncated)

Suitable functions could be $\pm\left(4e^{2x} - 2x - 68\right)$, $\pm\left(x - \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)\right)$, $\pm\left(2x - \ln\left(\frac{1}{2}x + 17\right)\right)$.

Using $4e^{2x} - 2x - 68$ f (1.4365) = -0.1, f (1.4375) = +0.02 or +0.03

Using $2e^{2x} - x - 34$ f (1.4365) = -0.05/-0.06, f (1.4375) = +0.01

Using $x - \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$ f (1.4365) =-0.0007 or -0.0008, f (1.4375) =+ 0.0001 or +0.0002

Using $2x - \ln\left(\frac{1}{2}x + 17\right)$ f (1.4365) = -0.001 or -0.002, f (1.4375) =+0.0003 or +0.0004

and states a reason (eg change of sign)

and a gives a minimal conclusion (eg root or tick)

It is valid to compare the two functions. Eg $g(1.4365) = 45.7(6) < 2\times 1.4365 + 43 = 45.8(73)$
$g(1.4375) = 45.90 > 2\times 1.4375 + 43 = 45.8(75)$

but the conclusion should be $g(x) = 2x + 43$ in between, hence root .

Similarly candidates can compare the functions x and $\frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$

June 2015 Mathematics Advanced Paper 1: Pure Mathematics 3

4.

Question Number	Scheme	Marks
6.(a)	$2^{x+1}-3=17-x \Rightarrow 2^{x+1}=20-x$	M1
	$(x+1)\ln 2=\ln(20-x) \Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
(b)	Sub $x_0=3$ into $x_{n+1}=\dfrac{\ln(20-x_n)}{\ln 2}-1, \Rightarrow$ $x_1=3.087$ (awrt)	M1A1
	$x_2=3.080,\ x_3=3.081$ (awrt)	A1
		(3)
(c)	$A=(3.1, 13.9)$ cao	M1,A1
		(2)
		(8 marks)

Question Number	Scheme	Marks
6.(a)Alt	$2^{x+1}-3=17-x \Rightarrow 2^{x}=\dfrac{20-x}{2}$	M1
	$x\ln 2=\ln\dfrac{20-x}{2} \Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
6.(a) **backwards**	$x=\dfrac{\ln(20-x)}{\ln 2}-1 \Rightarrow (x+1)\ln 2=\ln(20-x)$	M1
	$\Rightarrow 2^{x+1}=20-x$	dM1
	Hence $y=2^{x+1}-3$ meets $y=17-x$	A1*
		(3)

(a)

M1 Setting equations in x equal to each other and proceeding to make 2^{x+1} the subject

dM1 Take ln's or logs of both sides, use the power law and proceed to $x = ..$

A1* This is a given answer and all aspects must be correct including ln or $\log_e$ rather than $\log_{10}$

Bracketing on both $(x+1)$ and $\ln(20-x)$ must be correct.

Eg $x+1\ln 2 = \ln(20-x) \Rightarrow x = \dfrac{\ln(20-x)}{\ln 2} - 1$ is A0*

Special case: Students who start from the point $2^{x+1} = 20 - x$ can score M1 dM1A0*

(b)

M1 Sub $x_0 = 3$ into $x_{n+1} = \dfrac{\ln(20-x_n)}{\ln 2} - 1$ to find $x_1 = ..$

Accept as evidence $x_1 = \dfrac{\ln(20-3)}{\ln 2} - 1$, awrt $x_1 = 3.1$

Allow $x_0 = 3$ into the miscopied iterative equation $x_1 = \dfrac{\ln(20-3)}{\ln 2}$ to find $x_1 = ..$

Note that the answer to this, 4.087, on its own without sight of $\dfrac{\ln(20-3)}{\ln 2}$ is M0

A1 awrt 3 dp $x_1 = 3.087$

A1 awrt $x_2 = 3.080$, $x_3 = 3.081$. Tolerate 3.08 for 3.080

Note that the subscripts are not important, just mark in the order seen

(c) Note that this appears as B1B1 on e pen. It is marked M1A1

M1 For sight of 3.1

Alternatively it can be scored for substituting their value of x or a rounded value of x from (b) into either $2^{x+1} - 3$ or $17 - x$ to find the y coordinate.

A1 $(3.1, 13.9)$

June 2014 Mathematics Advanced Paper 1: Pure Mathematics 3

5.

Question Number	Scheme	Marks
6.(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign$\Rightarrow Q$ lies between	A1 (2)
(b)	At R $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$	A1 (2)
		(8 marks)

(a)

M1 Sub both $x = 2.1$ and $x = 2.2$ into y and achieve at least one correct to 1 sig fig

In radians $y_{2.1} = \text{awrt} -0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

In degrees $y_{2.1} = \text{awrt } 3$ $y_{2.2} = \text{awrt } 4$

A1 Both values correct to 1 sf with a reason and a minimal conclusion.

$y_{2.1} = \text{awrt} -0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

Accept change of sign, positive and negative, $y_{2.1} \times y_{2.2} = -1$ as reasons and hence root, Q lies between 2.1 and 2.2, QED as a minimal conclussion.

Accept a smaller interval spanning the root of 2.131528, say 2.13 and 2.14, but the A1 can only be scored when the candidate refers back to the question, stating that as root lies between 2.13 and 2.14 it lies between 2.1 and 2.2

(b)

M1 Differentiating to get $\frac{dy}{dx} = \ldots\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ where ... is a constant, or a linear function in x.

A1 $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$

M1 Sets their $\frac{dy}{dx}=0$ and proceeds to make the x of their $3x^2$ the subject of the formula

Alternatively they could state $\frac{dy}{dx}=0$ and write a line such as

$2x\sin\left(\frac{1}{2}x^2\right)=3x^2-3$, before making the x of $3x^2$ the subject of the formula

A1* Correct given solution. $x=\sqrt{1+\frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$

Watch for missing x's in their formula

(c)

M1 Subs x= 1.3 into the iterative formula to find at least x_1.

This can be implied by $x_1=\text{awrt}1.3$ (not just 1.3)

or $x_1=\sqrt{1+\frac{2}{3}\times1.3\sin\left(\frac{1}{2}\times1.3^2\right)}$ or $x_1=\text{awrt}1.006$ (degrees)

A1 Both answers correct (awrt 3 decimal places). The subscripts are not important. Mark as the first and second values seen. $x_1=\text{awrt }1.284$ $x_2=\text{awrt }1.276$

June 2013 Mathematics Advanced Paper 1: Pure Mathematics 3

6.

Question Number	Scheme	Marks
4(a)	$f'(x)=50x^2e^{2x}+50xe^{2x}$ oe.	M1A1
	Puts $f'(x)=0$ to give $x=-1$ and $x=0$ or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x}-16=0\Rightarrow x^2=\frac{16}{25}e^{-2x}\Rightarrow x=\pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs x_0= 0.5 into $x=\frac{4}{5}e^{-x}\Rightarrow x_1=\text{awrt }0.485$	M1A1
	$\Rightarrow x_2=\text{awrt }0.492,\ x_3=\text{awrt }0.489$	A1
		(3)
(d)	$\alpha=0.49$	B1
	$f(0.485)=-0.487,\ f(0.495)=(+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question 4

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu'+uv'$. If the rule is quoted it must be correct.

It can be implied by their $u=..,v=...,u'=...,v'=...$ followed by their $vu'+uv'$

If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x}+Bxe^{2x}$

A1 $f'(x)=50x^2e^{2x}+50xe^{2x}$.

Allow un simplified forms such as $f'(x)=25x^2\times 2e^{2x}+50x\times e^{2x}$

dM1 Sets $f'(x)=0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x

This is dependent upon the first M1 being scored.

A1 Both $x=-1$ and $x=0$ or one complete coordinate . Accept $(0,-16)$ and $(-1, 25e^{-2}-16)$ or $(-1, awrt\ -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.

$x=-1, 0$ $y=\frac{25}{e^2}-16, -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1, 25e^{-2}-16)$ but the 'pairs' must be correct and exact.

Notes for Question 4 Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x}-16=0$ or $25x^2e^{2x}=16$

2) Show at least one intermediate (correct) line with either

x^2 or x the subject. Eg $x^2=\frac{16}{25}e^{-2x}$, $x=\sqrt{\frac{16}{25}e^{-2x}}$ oe

or square rooting $25x^2e^{2x}=16 \Rightarrow 5xe^x=\pm 4$

or factorising by DOTS to give $(5xe^x+4)(5xe^x-4)=0$

3) Show the given answer $x=\pm\frac{4}{5}e^{-x}$.

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that f(x)=0

(c)

M1 Substitutes $x_0=0.5$ into $x=\frac{4}{5}e^{-x} \Rightarrow x_1=....$

This can be implied by $x_1=\frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 $x_1=$ awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript.

A1 $x_2=$ awrt 0.492, $x_3=$ awrt 0.489 3dp. Mark as the second and third values given.

(d)

B1 States $\alpha = 0.49$

B1 Justifies **by**

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$f(0.485) = -0.5$, $f(0.495) = (+)0.5$ rounded

$f(0.485) = -0.4$, $f(0.495) = (+)0.4$ truncated

giving a reason – accept change of sign, >0 <0 or $f(0.485) \times f(0.495) < 0$

and giving a minimal conclusion. Eg. Accept hence root or $\alpha = 0.49$

A smaller interval containing the root may be used, eg $f(0.49)$ and $f(0.495)$. Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of x_4= awrt 0.491 and stating (or seeing each value round to) 0.49

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 3

7.

Question Number	Scheme	Marks
2.	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x) + 1$	M1A1*
		(2)
	(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1 \Rightarrow x_1 = 2.3863$	M1, A1
	AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$	A1
		(3)
	(c) Chooses interval [2.3065,2.3075]	M1
	$g(2.3065)$=-0.0002(7), $g(2.3075)$=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1
		(3)
		(8 marks)

(a) M1 Sets g(x)=0,and using correct *ln* work, makes the x of the e^{x-1} term the subject of the formula.

Look for $e^{x-1} + x - 6 = 0 \Rightarrow e^{x-1} = \pm 6 \pm x \Rightarrow x = \ln(\pm 6 \pm x) \pm 1$

Do not accept $e^{x-1} = 6 - x$ **without firstly seeing** $e^{x-1} + x - 6 = 0$ **or a statement that g(x)=0 ⇒**

A1* cso. $x = \ln(6-x) + 1$ **Note that this is a given answer (and a proof).**

'Invisible' brackets are allowed for the M but not the A

Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.

$0 = e^{x-1} + x - 6 \Rightarrow \quad 0 = x - 1 + \ln(x-6) \Rightarrow x = \ln(6-x) + 1$

(b) M1 Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1$ to produce a numerical value for x_1.
Evidence for the award could be any of $\ln(6-2)+1$, $\ln 4 + 1$, 2.3….. or awrt 2.4

A1 Answer correct to 4 dp $x_1 = 2.3863$.
The subscript is not important. Mark as the first value given/found.

A1 Awrt 4 dp. $x_2 = 2.2847$ *and* $x_3 = 2.3125$
The subscripts are not important. Mark as the second and third values given/found

(c) M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates $g(2.3065)$ and $g(2.3075)$ with at least one of these correct to 1sf.
The answers can be rounded or truncated
$g(2.3065)$ = -0.0003 rounded, $g(2.3065)$= -0.0002 truncated
$g(2.3075)$ = (+) 0.004 rounded and truncated

A1 Both values correct (rounded or truncated),
A reason which could include change of sign, >0 <0, $g(2.3065) \times g(2.3075) < 0$
AND a minimal conclusion such as hence root, α=2.307 or □
Do not accept continued iteration as question demands an interval to be chosen.

Alternative solution to (a) working backwards

M1 Proceeds from $x = \ln(6 - x) + 1$ using correct exp work to …….=0

A1 **Arrives correctly** at $e^{x-1} + x - 6 = 0$ **and** makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $f(x) = \ln(6 - x) + 1 - x$ **{Similarly** $h(x) = x - 1 - \ln(6 - x)$ }

M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated
f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Even accept 0.0001

f(2.3075) = - 0.0011.. Accept -0.001 rounded or truncated

June 2012 Mathematics Advanced Paper 1: Pure Mathematics 3

8.

Question Number	Scheme	Marks
2.	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1A1*
		(3)
	(b) $x_1=1.41$, *awrt* $x_2=1.20$ $x_3=1.31$	M1A1,A1
		(3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	$\text{f}(1.2725)=(+)0.00827...$ $\text{f}(1.2715)=-0.00821....$	M1
	Change of sign$\Rightarrow\alpha$=1.272	A1
		(3)
		(9 marks)

Notes

(a) M1 Moves from f(x)=0, which may be implied by subsequent working, to $x^2(x\pm3)=\pm12\pm4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1 Divides by '(x+3)' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage

A1* CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12-4x needs to have been factorised.

(b) **Note that this appears B1,B1,B1 on EPEN**

M1 An attempt to substitute $x_0=1$ into the iterative formula to calculate x_1.

This can be awarded for the sight of $\sqrt{\dfrac{4(3-1)}{(3+1)}}$, $\sqrt{\dfrac{8}{4}}$, $\sqrt{2}$ and even 1.4

A1 $x_1=1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0

A1 $x_2=$ awrt 1.20 $x_3=$ awrt 1.31. Mark as the second and third values found. Condone 1.2 for x_2

(c) **Note that this appears M1A1A1 on EPEN**

M1 Choosing the interval (1.2715,1.2725) or tighter containing the root 1.271998323. Continued iteration is not allowed for this question and is M0

M1 Calculates f(1.2715) and f(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp
Accept f(1.2725) = (+)0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

Alternative to (a) working backwards

2(a)

	$x = \sqrt{\dfrac{4(3-x)}{(x+3)}} \Rightarrow x^2 = \dfrac{4(3-x)}{(x+3)} \Rightarrow x^2(x+3) = 4(3-x)$	M1	
	$x^3 + 3x^2 = 12 - 4x \Rightarrow x^3 + 3x^2 + 4x - 12 = 0$	dM1	
	States that this is f(x)=0	A1*	(3)

Alternative starting with the given result and working backwards

M1 Square (both sides) and multiply by (x+3)

dM1 Expand brackets and collect terms on one side of the equation =0

A1 A statement to the effect that this is f(x)=0

An acceptable answer to (c) with an example of a tighter interval

M1 Choosing the interval (1.2715, 1.2720). This contains the root 1.2719(98323)

M1 Calculates f(1.2715) and f(1.2720), with at least 1 correct to 1 sig fig rounded or truncated.
Accept f(1.2715) = -0.008 1sf rounded or truncated f(1.2715) = -0.01 2dp
Accept f(1.2720) = (+)0.00003 1sf rounded or f(1.2720) = (+)0.00002 truncated 1sf

A1 Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2720)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

x	f(x)
1.2715	-0.00821362
1.2716	-0.00656564
1.2717	-0.00491752
1.2718	-0.00326927
1.2719	-0.00162088
1.2720	+0.00002765
1.2721	+0.00167631
1.2722	+0.00332511
1.2723	+0.00497405
1.2724	+0.00662312
1.2725	+0.00827233

An acceptable answer to (c) using g(x) where g(x)= $\sqrt{\dfrac{4(3-x)}{(x+3)}} - x$

2nd M1 Calculates g(1.2715) and g(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated.
g(1.2715) = 0.0007559. Accept g(1.2715) = awrt (+)0.0008 1sf rounded or awrt 0.0007 truncated.
g(1.2725)=-0.00076105. Accept g(1.2725) = awrt -0.0008 1sf rounded or awrt -0.0007 truncated.

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 3

9.

Question No	Scheme	Marks
6	(a) $f(0.8) = 0.082$, $f(0.9) = -0.089$	M1
	Change of sign $\Rightarrow$ root (0.8,0.9)	A1
		(2)
	(b)	
	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$	M1A1*
		(4)
	(c) Sub $x_0=2$ into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$	M1
	x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	A1,A1
		(3)
	(d) [1.90775,1.90785]	M1
	f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076..	M1
	Change of sign $\Rightarrow$ x=1.9078	A1
		(3)
		(12 marks)

(a)

M1 Calculates both f(0.8) and f(0.9). Evidence of this mark could be, either, seeing both '*x*' substitutions written out in the expression, or, one value correct to 1 sig fig, or the appearance of incorrect values of f(0.8)=awrt 0.2 or f(0.9)=awrt 0.1 from use of degrees

A1 This requires both values to be correct as well as a reason and a conclusion.
Accept f(0.8)= 0.08 truncated or rounded (2dp) or 0.1 rounded (1dp) and f(0.9)=-0.08 truncated or rounded as -0.09 (2dp) or -0.1(1dp)
Acceptable reasons are change of sign, <0 >0, +ve –ve, f(0.8)f(0.9)<0. Acceptable conclusion is hence root or □

(b)

M1 Attempts to differentiate f(x). Seeing any of 2x,3 or ±Asin(½x) is sufficient evidence.

A1 f'(x) correct. Accept $\frac{dy}{dx} = 2x - 3 - \sin\left(\frac{1}{2}x\right)$

M1 Sets their f'(x)=0 and proceeds to x=….. You must be sure that they are setting what they think is f'(x)=0.
Accept $2x = 3 + \sin\left(\frac{1}{2}x\right)$ going to x=..only if f'(x) =0 is stated first

A1 * $x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$. This is a given answer so don't accept just the sight of this answer. It is cso

(c) M1 Substitutes $x_0=2$ into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$. Evidence of this mark could be awrt 1.9 or 1.5 (from degrees)

A1 x_1=awrt 1.921

A1 x_2=awrt 1.91(0) and x_3=awrt 1.908

(d) **Continued iteration is not acceptable for this part. Question states 'By choosing a suitable interval…'**

M1 Chooses the interval [1.90775,1.90785] or tighter containing the root= 1.907845522

M1 Calculates f'(1.90775) and f'(1.90785) or tighter with at least one correct, rounded or truncated
f'(1.90775)=-0.0001 truncated or awrt -0.0002 rounded
f'(1.90785)= 0.000007 truncated or awrt 0.000008 rounded

Accept versions of g(*x*)-*x* where g(*x*) $=\frac{3+\sin\left(\frac{1}{2}x\right)}{2}$.

When x= 1.90775, $g(x) - x = 8 \times 10^{-5}$ rounded and truncated
When x= 1.90785, $g(x) - x = -3 \times 10^{-6}$ truncated or $= -4 \times 10^{-6}$ rounded

A1 Both values correct, rounded or truncated, a valid reason (see part a) and a minimal conclusion (see part a). Saying hence root is acceptable. There is no need to refer to the 'turning point'.

June 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

10.

2 (a)	f(0.75)= -0.18… f(0.85) = 0.17…..	M1
	Change of sign, hence root between x=0.75 and x=0.85	A1 (2)
(b)	Sub x_0=0.8 into $x_{n+1} = [\arcsin(1-0.5x_n)]^{\frac{1}{2}}$ to obtain x_1	M1
	Awrt x_1=0.80219 and x_2=0.80133	A1
	Awrt x_3 = 0.80167	A1 (3)
(c)	f(0.801565) = -2.7…..$\times 10^{-5}$ f(0.801575) = +8.6….$\times 10^{-6}$	M1A1
	Change of sign and conclusion	A1 (3)
	See Notes for continued iteration method	
		8 Marks

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

11.

Question Number	Scheme		Marks
5. **(a)**	Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8-x)\ln x = 0$		
	Either $(8-x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of $\{x\}=1$ OR $x=\{8\}$	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
(b)	Apply product rule: $\left\{\begin{matrix} u = (8-x) & v = \ln x \\ \frac{du}{dx} = -1 & \frac{dv}{dx} = \frac{1}{x} \end{matrix}\right\}$	$vu' + uv'$	M1
	$f'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
(c)	$f'(3.5) = 0.032951317...$ $f'(3.6) = -0.058711623...$ Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $f'(3.5)$ and $f'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
(d)	At Q, $f'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $f'(x) = 0$.	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to x=	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question Number	Scheme		Marks
(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55)+1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$ $x_2 = 3.538246011...$ $x_3 = 3.534144722...$	Both x_1 = awrt 3.529 and x_2 = awrt 3.538	A1
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534,$ to 3 dp.	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3) **[13]**

June 2010 Mathematics Advanced Paper 1: Pure Mathematics 3

12.

Question Number	Scheme	Marks
3. (a)	$f(1.2) = 0.49166551...,\quad f(1.3) = -0.048719817...$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that $\alpha \in [1.2, 1.3]$	M1A1 (2)
(b)	$4\operatorname{cosec}x - 4x + 1 = 0 \Rightarrow 4x = 4\operatorname{cosec}x + 1$	M1
	$\Rightarrow x = \operatorname{cosec}x + \dfrac{1}{4} \Rightarrow \underline{x = \dfrac{1}{\sin x} + \dfrac{1}{4}}$	A1 * (2)
(c)	$x_1 = \dfrac{1}{\sin(1.25)} + \dfrac{1}{4}$	M1
	$x_1 = 1.303757858...,\quad x_2 = 1.286745793...$	A1
	$x_3 = 1.291744613...$	A1 (3)
(d)	$f(1.2905) = 0.00044566695...,\quad f(1.2915) = -0.00475017278...$ Sign change (and as $f(x)$ is continuous) therefore a root α is such that	M1
	$\alpha \in (1.2905, 1.2915) \Rightarrow \alpha = 1.291$ (3 dp)	A1 (2) **[9]**

	(a) M1: Attempts to evaluate both $f(1.2)$ and $f(1.3)$ and evaluates at least one of them correctly to awrt (or truncated) 1 sf. A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion. (b) M1: Attempt to make $4x$ or x the subject of the equation. A1: Candidate must then rearrange the equation to give the required result. It must be clear that candidate has made their initial $f(x) = 0$. (c) M1: An attempt to substitute $x_0 = 1.25$ into the iterative formula Eg $= \frac{1}{\sin(1.25)} + \frac{1}{4}$. Can be implied by $x_1 =$ awrt 1.3 or $x_1 =$ awrt 46°. A1: Both $x_1 =$ awrt 1.3038 and $x_2 =$ awrt 1.2867 A1: $x_3 =$ awrt 1.2917 (d) M1: Choose suitable interval for x, e.g. [1.2905, 1.2915] or tighter and at least one attempt to evaluate f(x). A1: both values correct to awrt (or truncated) 1 sf, sign change and conclusion.	

Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 3

13.

Question Number	Scheme		Marks
Q2	$f(x) = x^3 + 2x^2 - 3x - 11$		
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$	Sets $f(x) = 0$ (can be implied) and takes out a factor of x^2 from $x^3 + 2x^2$, or x from $x^3 + 2x$ (slip).	M1
	$\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x+11}{x+2}$ $\Rightarrow x = \sqrt{\left(\frac{3x+11}{x+2}\right)}$	then rearranges to give the quoted result on the question paper.	A1 AG
			(2)
(b)	Iterative formula: $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$, $x_1 = 0$		
	$x_2 = \sqrt{\left(\frac{3(0)+11}{(0)+2}\right)}$	An attempt to substitute $x_1 = 0$ into the iterative formula. Can be implied by $x_2 = \sqrt{5.5}$ ***or 2.35*** or awrt 2.345	M1
	$x_2 = 2.34520788...$ $x_3 = 2.037324945...$	Both $x_2 =$ awrt 2.345 and $x_3 =$ awrt 2.037	A1
	$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1
			(3)

(c)	Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$		
	$f(2.0565) = -0.013781637...$ $f(2.0575) = 0.0041401094...$	Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter	M1
	Sign change (and $f(x)$ is continuous) therefore a root α is	any one value awrt 1 sf	dM1
	such that $\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057$ (3 dp)	both values correct awrt 1sf, sign change and conclusion	A1
			(3)
		As a minimum, both values must be correct to 1 sf, candidate states "change of sign, hence root".	
			[8]