# Vectors – Mark Scheme

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 4

1.

| Question Number | Scheme | Notes | Marks |
| --- | --- | --- | --- |
| **6.** | $l_1 : \mathbf{r} = \begin{pmatrix}4\\28\\4\end{pmatrix} + \lambda\begin{pmatrix}-1\\-5\\1\end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix}5\\3\\1\end{pmatrix} + \mu\begin{pmatrix}3\\0\\-4\end{pmatrix}$ ; $\overrightarrow{OA} = \begin{pmatrix}2\\18\\6\end{pmatrix}$ lies on $l_1$ | Let $\theta_{\text{Acute}}$ be the acute angle between $l_1$ and $l_2$ | |
| (a) | $\{l_1 = l_2 \Rightarrow\}\ 28 - 5\lambda = 3\ \{\Rightarrow \lambda = 5\}$ or $4-\lambda = 5+3\mu$ and $4+\lambda = 1-4\mu\ \{\Rightarrow \mu = -2\}$ | $28-5\lambda = 3$ or $4-\lambda = 5+3\mu$ and $4+\lambda = 1-4\mu$ or $\lambda = 5$ or $\mu = -2$ (Can be implied). | B1 |
| | $\{\overrightarrow{OX} =\}\ \begin{pmatrix}4\\28\\4\end{pmatrix} + 5\begin{pmatrix}-1\\-5\\1\end{pmatrix}$ or $\begin{pmatrix}5\\3\\1\end{pmatrix} - 2\begin{pmatrix}3\\0\\-4\end{pmatrix}$ | Puts $l_1 = l_2$ and solves to find $\lambda$ and/or $\mu$ **and** substitutes their value for $\lambda$ into $l_1$ or their value for $\mu$ into $l_2$ | M1 |
| | So, $X(-1, 3, 9)$ | $(-1, 3, 9)$ or $\begin{pmatrix}-1\\3\\9\end{pmatrix}$ or $-\mathbf{i}+3\mathbf{j}+9\mathbf{k}$ or condone $\begin{matrix}-1\\3\\9\end{matrix}$ | A1 **cao** |
| | | | **[3]** |
| (b) **Way 1** | $\mathbf{d}_1 = \begin{pmatrix}-1\\-5\\1\end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix}3\\0\\-4\end{pmatrix} \Rightarrow \begin{pmatrix}-1\\-5\\1\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-4\end{pmatrix}$ | Realisation that the dot product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$ | M1 |
| | $\cos\theta = \dfrac{\pm\begin{pmatrix}-1\\-5\\1\end{pmatrix} \bullet \begin{pmatrix}3\\0\\-4\end{pmatrix}}{\sqrt{(-1)^2+(-5)^2+(1)^2}\cdot\sqrt{(3)^2+(0)^2+(-4)^2}}\ \left\{= \dfrac{-7}{\sqrt{27}.\sqrt{25}}\right\}$ | **dependent on the 1st M mark.** Applies dot product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$ | dM1 |
| | $\{\theta = 105.6303588... \Rightarrow\}\ \theta_{\text{Acute}} = 74.36964117... = 74.37$ (2 dp) | awrt 74.37 seen in (b) only | A1 |
| | | | **[3]** |
| (c) | $\overrightarrow{AX} = "\overrightarrow{OX}" - \overrightarrow{OA} = \begin{pmatrix}-1\\3\\9\end{pmatrix} - \begin{pmatrix}2\\18\\6\end{pmatrix} = \begin{pmatrix}-3\\-15\\3\end{pmatrix}$ or $A_{\lambda=2}, X_{\lambda=5} \Rightarrow AX = 3\lvert\mathbf{d}_1\rvert$, $\{\lvert\mathbf{d}_1\rvert = \sqrt{27}\}$ | | |
| | $AX = \sqrt{(-3)^2+(-15)^2+(3)^2}$ or $3\sqrt{27}\ \{= \sqrt{243}\} = 9\sqrt{3}$ | Full method for finding $AX$ or $XA$ | M1 |
| | | $9\sqrt{3}$ seen in (c) only | A1 **cao** |
| | **Note:** You cannot recover work for part (c) in either part (d) or part (e). | | **[2]** |
| (d) **Way 1** | $\dfrac{YA}{"9\sqrt{3}"} = \tan("74.36964...")$ | $\dfrac{YA}{\text{their }\lvert AX\rvert} = \tan\theta$ or $YA = (\text{their }\lvert AX\rvert)\tan\theta$, where $\theta$ is their acute or obtuse angle between $l_1$ and $l_2$ | M1 |
| | $YA = 55.71758... = 55.7$ (1 dp) | anything that rounds to 55.7 | A1 |
| | | | **[2]** |

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| (e) **Way 1** | $\{A_{\lambda=2}, X_{\lambda=5} \Rightarrow \text{So } AX = 2AB \Rightarrow \text{So at } B,\ \lambda = 3.5 \text{ or } \lambda = 0.5)\}$ | | |
| | $\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 3.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$ | Substitutes **either** $\lambda = \dfrac{(\text{their } \lambda_X \text{ found in } (a)) + 2}{2}$ **or** $\lambda_\beta = 3 - \dfrac{(\text{their } \lambda_X \text{ found in } (a))}{2}$ into $l_1$ | M1; |
| | $\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 0.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$ | At least one position vector is correct. (Also allow coordinates). | A1 |
| | | Both position vectors are correct. (Also allow coordinates). | A1 |
| | | | **[3]** |
| | | | **13** |

| Question Number | Scheme | Notes | Marks |
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| **6.** (e) **Way 2** | $\left\{AX = 2AB \Rightarrow AB = \tfrac{1}{2}AX.\ \text{So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \overrightarrow{AB} \Rightarrow \overrightarrow{OB} = \overrightarrow{OA} \pm \tfrac{1}{2}\overrightarrow{AX}\right\}$ | | |
| | $\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} + 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$ | Applies either $\overrightarrow{OA} + 0.5\overrightarrow{AX}$ **or** $\overrightarrow{OA} - 0.5\overrightarrow{AX}$ where $(\text{their } \overrightarrow{AX}) = \pm\left[(\text{their } \overrightarrow{OX}) - \overrightarrow{OA}\right]$ | M1; |
| | $\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$ | At least one position vector is correct (Also allow coordinates) | A1 |
| | | Both position vectors are correct (Also allow coordinates) | A1 |
| | | | **[3]** |
| **6.** (e) **Way 3** | $\overrightarrow{AB} = \begin{pmatrix}4-\lambda\\28-5\lambda\\4+\lambda\end{pmatrix} - \begin{pmatrix}2\\18\\6\end{pmatrix} = \begin{pmatrix}2-\lambda\\10-5\lambda\\-2+\lambda\end{pmatrix} = \begin{pmatrix}1(2-\lambda)\\5(2-\lambda)\\-1(2-\lambda)\end{pmatrix};\ \overrightarrow{AX} = \begin{pmatrix}-3\\-15\\3\end{pmatrix}$ $\quad AX^2 = 243 \Rightarrow AB^2 = 27(2-\lambda)^2$ <br> $AX = 2AB \Rightarrow AX^2 = 4AB^2 \Rightarrow 243 = 4(27)(2-\lambda)^2 \Rightarrow (2-\lambda)^2 = \frac{9}{4}$ **or** $27\lambda^2 - 108\lambda + \frac{189}{4} = 0$ <br> **or** $108\lambda^2 - 432\lambda + 189 = 0$ **or** $4\lambda^2 - 16\lambda + 7 = 0 \Rightarrow \lambda = 3.5 \text{ or } \lambda = 0.5$ | | |
| | $\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 3.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$ | Full method of solving for $\lambda$ the equation $AX^2 = 4AB^2$ using (their $\overrightarrow{AX}$) and $\overrightarrow{AB}$ **and** substitutes at least one of their values for $\lambda$ into $l_1$ | M1; |
| | $\overrightarrow{OB} = \begin{pmatrix}4\\28\\4\end{pmatrix} + 0.5\begin{pmatrix}-1\\-5\\1\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$ | At least one position vector is correct (Also allow coordinates) | A1 |
| | | Both position vectors are correct (Also allow coordinates) | A1 |
| | **Note:** $AX = 2AB \Rightarrow \overrightarrow{AX} = \pm 2\overrightarrow{AB}$. Hence, $\lambda = 3.5$ or $\lambda = 0.5$ can be found from solving either $x: -3 = \pm 2(2-\lambda)$ or $y: -15 = \pm 2(10 - 5\lambda)$ or $z: -3 = \pm 2(-2+\lambda)$ | | **[3]** |

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| **6.** (e) **Way 4** | $\overrightarrow{OB} = \begin{pmatrix}-1\\3\\9\end{pmatrix} + 0.5\begin{pmatrix}3\\15\\-3\end{pmatrix}; = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$ | Applies **either** (their $\overrightarrow{OX}$) $+0.5\overrightarrow{XA}$ **or** (their $\overrightarrow{OX}$) $+1.5\overrightarrow{XA}$ where (their $\overrightarrow{XA}$) $= \overrightarrow{OA} -$ (their $\overrightarrow{OX}$) | M1; |
| | $\overrightarrow{OB} = \begin{pmatrix}-1\\3\\9\end{pmatrix} + 1.5\begin{pmatrix}3\\15\\-3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$ | At least one position vector is correct (Also allow coordinates) | A1 |
| | | Both position vectors are correct (Also allow coordinates) | A1 |
| | | | **[3]** |
| **6.** (e) **Way 5** | $\overrightarrow{OB} = 0.5\left(\begin{pmatrix}-1\\3\\9\end{pmatrix} + \begin{pmatrix}2\\18\\6\end{pmatrix}\right); = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$ | Applies $\frac{1}{2}\left[(\text{their } \overrightarrow{OX}) + \overrightarrow{OA}\right]$ | M1; |
| | $\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\begin{pmatrix}-3\\-15\\3\end{pmatrix}; = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$ | At least one position vector is correct (Also allow coordinates) | A1 |
| | | Both position vectors are correct (Also allow coordinates) | A1 |
| | | | **[3]** |

| Question Number | Scheme | Notes | Marks |
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| **6.** (e) **Way 6** | $\left\{\left\|\overrightarrow{AX}\right\| = 9\sqrt{3}, \left\|d_1\right\| = 3\sqrt{3} \Rightarrow K = \frac{9\sqrt{3}}{3\sqrt{3}} = 3 \Rightarrow \overrightarrow{AX} = 3\mathbf{d}_1; \text{So, } \overrightarrow{OB} = \overrightarrow{OA} \pm \tfrac{1}{2}\overrightarrow{AX} = \overrightarrow{OA} \pm \tfrac{1}{2}(3\mathbf{d}_1)\right\}$ | | |
| | $\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} + 0.5\left(3\begin{pmatrix}-1\\-5\\1\end{pmatrix}\right); = \begin{pmatrix}0.5\\10.5\\7.5\end{pmatrix}$ | Applies either $\overrightarrow{OA} + 0.5(K\mathbf{d}_1)$ **or** $\overrightarrow{OA} - 0.5(K\mathbf{d}_1)$, where $K = \frac{\text{their } \left\|\overrightarrow{AX}\right\|}{3\sqrt{3}}$ | M1; |
| | $\overrightarrow{OB} = \begin{pmatrix}2\\18\\6\end{pmatrix} - 0.5\left(3\begin{pmatrix}-1\\-5\\1\end{pmatrix}\right); = \begin{pmatrix}3.5\\25.5\\4.5\end{pmatrix}$ | At least one position vector is correct (Also allow coordinates) | A1 |
| | | Both position vectors are correct (Also allow coordinates) | A1 |
| | | | **[3]** |

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| | | **Question 6 Notes** |
| **6.** (a) | **Note** | M1 can be implied by at least two correct follow through coordinates from their $\lambda$ or from their $\mu$ |
| (b) | **Note** | ***Evaluating*** the dot product (i.e. $(-1)(3) + (-5)(0) + (1)(-4)$) is not required for the M1, dM1 marks. |
| | **Note** | **For M1 dM1:** Allow one slip in writing down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$ |
| | **Note** | Allow M1 dM1 for $\left(\sqrt{(-1)^2+(-5)^2+(1)^2}\,.\sqrt{(3)^2+(0)^2+(-4)^2}\right)\cos\theta = \pm\begin{pmatrix}-1\\-5\\1\end{pmatrix}\langle\begin{pmatrix}3\\0\\-4\end{pmatrix}$ |
| | **Note** | $\theta = 1.297995...^c$, (without evidence of awrt 74.37) is A0 |

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| **6.** (b) **Way 2** | **Alternative Method: Vector Cross Product** | | |
| | **Only apply this scheme if it is clear that a vector cross product method is being applied.** | | |
| | $\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -5 & 1 \\ 3 & 0 & -4 \end{vmatrix} = 20\mathbf{i} - \mathbf{j} + 15\mathbf{k} \right\}$ | Realisation that the vector cross product is required between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$ | M1 |
| | $\sin\theta = \dfrac{\sqrt{(20)^2+(-1)^2+(15)^2}}{\sqrt{(-1)^2+(-5)^2+(1)^2}\,.\sqrt{(3)^2+(0)^2+(-4)^2}}$ | Applies the vector product formula between $\mathbf{d}_1$ and $\mathbf{d}_2$ or a multiple of $\mathbf{d}_1$ and $\mathbf{d}_2$ | dM1 |
| | $\sin\theta = \dfrac{\sqrt{626}}{\sqrt{27}.\sqrt{25}} \Rightarrow \theta = 74.36964117... = 74.37$ (2 dp) | awrt 74.37 seen in (b) only | A1 |
| | | | **[3]** |

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|---|---|---|
| **6.** (c) | **M1** | Finds the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ and applies Pythagoras to the result to find $AX$ or $XA$ **OR** applies $\left|(\text{their } \lambda_X \text{ found in } (a)) - 2\right|.\sqrt{(-1)^2+(-5)^2+(1)^2}$ |
| | **Note** | For M1: Allow one slip in writing down their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ |
| | **Note** | Allow M1A1 for $\begin{pmatrix} 3 \\ 15 \\ 3 \end{pmatrix}$ leading to $AX = \sqrt{(3)^2+(15)^2+(3)^2} = \sqrt{243} = 9\sqrt{3}$ |
| (e) | **Note** | Imply M1 for no working leading to any two components of one of the $\overrightarrow{OB}$ which are correct. |

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|---|---|---|---|
| **6.** (d) **Way 2** | $\dfrac{"9\sqrt{3}"}{YA} = \tan(90 - "74.36964...")$ | $\dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{YA} = \tan(90-\theta)$ or $AY = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{\tan(90-\theta)}$, where $\theta$ is the acute or obtuse angle between $l_1$ and $l_2$ | M1 |
| | $YA = 55.71758... = 55.7$ (1 dp) | anything that rounds to 55.7 | A1 |
| | | | **[2]** |
| **6.** (d) **Way 3** | $\dfrac{YA}{\sin("74.36964...")} = \dfrac{"9\sqrt{3}"}{\sin(90 - "74.36964...")}$ | $\dfrac{YA}{\sin\theta} = \dfrac{\text{their } \lvert\overrightarrow{AX}\rvert}{\sin(90-\theta)}$ o.e., where $\theta$ is the acute or obtuse angle between $l_1$ and $l_2$ | M1 |
| | $YA = \dfrac{9\sqrt{3}\sin(74.36964...)}{\sin(15.63036...)} = 55.71758... = 55.7$ (1 dp) | anything that rounds to 55.7 | A1 |
| | | | **[2]** |

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| **6. (d) Way 4** | $\mathbf{d}_1 = \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}$, $\overrightarrow{OY} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix}$ | | |
| | $\overrightarrow{YA} = \begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix} - \begin{pmatrix} 5+3\mu \\ 3 \\ 1-4\mu \end{pmatrix} = \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix}$<br>$\overrightarrow{YA} \bullet \mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix} -3-3\mu \\ 15 \\ 5+4\mu \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix} = 0$<br>$\Rightarrow 3+3\mu-75+5+4\mu=0 \Rightarrow \mu = \frac{67}{7}$<br>$YA^2 = \left(-3-3\left(\frac{67}{7}\right)\right)^2 + (15)^2 + \left(5+4\left(\frac{67}{7}\right)\right)^2$ | (Allow a sign slip in copying $\mathbf{d}_1$)<br>Applies $\overrightarrow{YA} \bullet \mathbf{d}_1 = 0$ or $\overrightarrow{AY} \bullet \mathbf{d}_1 = 0$ or $\overrightarrow{YA} \bullet (K\mathbf{d}_1) = 0$ or $\overrightarrow{AY} \bullet (K\mathbf{d}_1) = 0$ to find $\mu$ and applies Pythagoras to find a numerical expression for $AY^2$ or for the distance $AY$ | M1 |
| | So, $YA = \sqrt{\left(-\frac{222}{7}\right)^2 + (15)^2 + \left(\frac{303}{7}\right)^2}$ | | |
| | $= 55.71758... = 55.7$ (1 dp) | anything that rounds to 55.7 | A1 |
| | **Note:** $\overrightarrow{OY} = \frac{236}{7}\mathbf{i} + 3\mathbf{j} - \frac{261}{7}\mathbf{k}$, $\overrightarrow{AY} = -\frac{222}{7}\mathbf{i} + 15\mathbf{j} + \frac{303}{7}\mathbf{k}$ | | **[2]** |

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2.

| Question Number | Scheme | Notes | Marks |
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| **8.** | $l_1 : \mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$. $\overrightarrow{OA}$ occurs when $\mu = 1$. $\overrightarrow{OP} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ | | |
| (a) | $A(3, 5, 0)$ | $(3, 5, 0)$ | B1 |
| | | | **[1]** |
| (b) | $\{l_2 :\}\ \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$ | $\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$, $\mathbf{a} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ | M1 |
| | | Correct vector equation using $\mathbf{r} =$ or $l =$ or $l_2 =$ | A1 |
| | $\mathbf{d}_2$ is the direction vector of $l_2$ | Do not allow $l_2 :$ or $l_2 \rightarrow$ or $l_1 =$ for the A1 mark. | **[2]** |
| (c) | $\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ | | |
| | $AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$ | Full method for finding $AP$ | M1 |
| | | $2\sqrt{2}$ | A1 |
| | | | **[2]** |

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| (d) | So $\overrightarrow{AP} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow \begin{pmatrix}-2\\0\\2\end{pmatrix} \bullet \begin{pmatrix}-5\\4\\3\end{pmatrix}$ | Realisation that the dot product is required between $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | M1 |
| | $\{\cos\theta =\}\ \dfrac{\overrightarrow{AP} \bullet \mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert . \lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix}-2\\0\\2\end{pmatrix} \bullet \begin{pmatrix}-5\\4\\3\end{pmatrix}\right)}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$ | **dependent on the previous M mark.** Applies dot product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | dM1 |
| | $\{\cos\theta\} = \dfrac{\pm(10+0+6)}{\sqrt{8}.\sqrt{50}} = \dfrac{4}{5}$ | $\{\cos\theta\} = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$ | A1 cso |
| | | | **[3]** |
| (e) | $\{\text{Area } APE =\}\ \frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ | $\frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin\theta$ or $\frac{1}{2}(\text{their } 2\sqrt{2})^2 \sin(\text{their } \theta)$ | M1 |
| | $= 2.4$ | 2.4 or $\dfrac{12}{5}$ or $\dfrac{24}{10}$ or awrt 2.40 | A1 |
| | | | **[2]** |
| (f) | $\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c) | | |
| | $\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$ | This mark can be implied. | M1 |
| | $\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \dfrac{4}{25} \Rightarrow\right\}\ \lambda = \pm\dfrac{2}{5}$ | Either $\lambda = \dfrac{2}{5}$ or $\lambda = -\dfrac{2}{5}$ | A1 |
| | $l_2 : \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} \pm \dfrac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ | **dependent on the previous M mark** Substitutes at least one of their values of $\lambda$ into $l_2$. | dM1 |
| | $\{\overrightarrow{OE}\} = \begin{pmatrix}3\\ \frac{17}{5}\\ \frac{4}{5}\end{pmatrix}$ or $\begin{pmatrix}3\\3.4\\0.8\end{pmatrix}$, $\{\overrightarrow{OE}\} = \begin{pmatrix}-1\\ \frac{33}{5}\\ \frac{16}{5}\end{pmatrix}$ or $\begin{pmatrix}-1\\6.6\\3.2\end{pmatrix}$ | At least one set of coordinates are correct. | A1 |
| | | Both sets of coordinates are correct. | A1 |
| | | | **[5]** |
| | | | **15** |

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| | | **Question 8 Notes** |
| **8.** (a) | **B1** | Allow $A(3, 5, 0)$ or $3\mathbf{i} + 5\mathbf{j}$ or $3\mathbf{i} + 5\mathbf{j} + 0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$ |
| (b) | **A1** | Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 = <br> i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$. |
| | **Note** | Allow the use of parameters $\mu$ or $t$ instead of $\lambda$. |
| (c) | **M1** | Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find $AP$ |
| | **Note** | Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$. |

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| (d) | Note | For both the M1 and dM1 marks $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) must be the vector used in part (c) or the difference $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ from part (a). |
| | Note | Applying the dot product formula correctly without $\cos\theta$ as the subject is fine for M1dM1 |
| | Note | ***Evaluating*** the dot product (i.e. $(-2)(-5)+(0)(4)+(2)(3)$) is not required for M1 and dM1 marks. |
| | Note | In part (d) allow one slip in writing $\overrightarrow{AP}$ and $\mathbf{d}_2$ |
| | Note | $\cos\theta = \dfrac{-10+0-6}{\sqrt{8}.\sqrt{50}} = -\dfrac{4}{5}$ followed by $\cos\theta = \dfrac{4}{5}$ is fine for A1 cso |
| | Note | Give M1dM1A1 for $\{\cos\theta =\} = \dfrac{\begin{pmatrix}-2\\0\\2\end{pmatrix}\bullet\begin{pmatrix}-10\\8\\6\end{pmatrix}}{\sqrt{8}.10\sqrt{2}} = \dfrac{20+12}{40} = \dfrac{4}{5}$ |
| | Note | Allow final A1 (ignore subsequent working) for $\cos\theta = 0.8$ followed by $36.869...^\circ$ |

***Alternative Method: Vector Cross Product***

**Only apply this scheme if it is clear that a candidate is applying a vector cross product method.**

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|---|---|---|
| $\overrightarrow{AP}\times\mathbf{d}_2 = \begin{pmatrix}-2\\0\\2\end{pmatrix}\times\begin{pmatrix}-5\\4\\3\end{pmatrix} = \left\{\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 0 & 2\\-5 & 4 & 3\end{vmatrix} = -8\mathbf{i}-4\mathbf{j}-8\mathbf{k}\right\}$ | Realisation that the vector cross product is required between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | M1 |
| $\sin\theta = \dfrac{\sqrt{(-8)^2+(-4)^2+(-8)^2}}{\sqrt{(-2)^2+(0)^2+(2)^2}.\sqrt{(-5)^2+(4)^2+(3)^2}}$ | Applies the vector product formula between their $(\overrightarrow{AP}$ or $\overrightarrow{PA})$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$ | dM1 |
| $\sin\theta = \dfrac{12}{\sqrt{8}.\sqrt{50}} = \dfrac{3}{5} \Rightarrow \cos\theta = \dfrac{4}{5}$ | $\cos\theta = \dfrac{4}{5}$ or 0.8 or $\dfrac{8}{10}$ or $\dfrac{16}{20}$ | A1 |

| | | |
|---|---|---|
| (e) | Note | Allow M1;A1 for $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^\circ - 36.869...^\circ)$; = awrt 2.40 |
| | Note | Candidates must use their $\theta$ from part (d) or apply a correct method of finding their $\sin\theta = \dfrac{3}{5}$ from their $\cos\theta = \dfrac{4}{5}$ |

| | | |
|---|---|---|
| | **Question 8 Notes Continued** | |
| **8.** (f) | Note | Allow the first M1A1 for deducing $\lambda = \dfrac{2}{5}$ or $\lambda = -\dfrac{2}{5}$ from no incorrect working |
| | SC | Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working |
| | Note | Give 1st M1 for $\sqrt{(-5\lambda)^2+(4\lambda)^2+(3\lambda)^2} = (\text{their } 2\sqrt{2})$ |
| | Note | Give 1st M0 for $(-5\lambda)^2+(4\lambda)^2+(3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent |
| | Note | Give 1st M1 for $\lambda = \dfrac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2+(4)^2+(3)^2}}$ and 1st A1 for $\lambda = \dfrac{2\sqrt{2}}{5\sqrt{2}}$ |
| | Note | So $\left\{\hat{\mathbf{d}}_1 = \dfrac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\}$ "vector" $= \dfrac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1 |
| | Note | The 2nd dM1 in part (f) can be implied for at least 2 (out of 6) correct $x, y, z$ ordinates from their values of $\lambda$. |
| | Note | Giving their "coordinates" as a column vector or position vector is fine for the final A1A1. |

| | | | |
|---|---|---|---|
| | CAREFUL | Putting $l_2$ equal to $A$ gives $\begin{pmatrix}1\\5\\2\end{pmatrix}+\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}=\begin{pmatrix}3\\5\\0\end{pmatrix}\rightarrow\begin{pmatrix}\lambda=\frac{2}{5}\\\lambda=0\\\lambda=-\frac{2}{3}\end{pmatrix}$ | Give M0 dM0 for finding and using $\lambda=\frac{2}{5}$ from this incorrect method. |
| | CAREFUL | Putting $\lambda\mathbf{d}_2=\overrightarrow{AP}$ gives $\lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}=\begin{pmatrix}2\\0\\-2\end{pmatrix}\rightarrow\begin{pmatrix}\lambda=-\frac{2}{5}\\\lambda=0\\\lambda=-\frac{2}{3}\end{pmatrix}$ | Give M0 dM0 for finding and using $\lambda=-\frac{2}{5}$ from this incorrect method. |
| | General | You can follow through the part (c) answer of their $AP=2\sqrt{2}$ for (d) M1dM1, (e) M1, (f) M1dM1 | |
| | General | You can follow through their $\mathbf{d}_2$ in part (b) for (d) M1dM1, (f) M1dM1. | |

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3.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 4. | $l_1:\mathbf{r}=\begin{pmatrix}5\\-3\\p\end{pmatrix}+\lambda\begin{pmatrix}0\\1\\-3\end{pmatrix}$, $l_2:\mathbf{r}=\begin{pmatrix}8\\5\\-2\end{pmatrix}+\mu\begin{pmatrix}3\\4\\-5\end{pmatrix}$. Let $\theta$ = acute angle between $l_1$ and $l_2$. | | |
| | **Note: You can mark parts (a) and (b) together.** | | |
| (a) | $\{l_1=l_2\Rightarrow\mathbf{i}:\}\ 5=8+3\mu\Rightarrow\mu=-1$ | Finds $\mu$ and substitutes their $\mu$ into $l_2$ | M1 |
| | So, $\{\overrightarrow{OA}\}=\begin{pmatrix}8\\5\\-2\end{pmatrix}-1\begin{pmatrix}3\\4\\-5\end{pmatrix}=\begin{pmatrix}5\\1\\3\end{pmatrix}$ | $5\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $\begin{pmatrix}5\\1\\3\end{pmatrix}$ or $(5, 1, 3)$ | A1 |
| | | | **[2]** |
| (b) | $\{\mathbf{j}:\ -3+\lambda=5+4\mu\Rightarrow\}\ -3+\lambda=5+4(-1)\Rightarrow\lambda=4$ | Equates **j** components, substitutes their $\mu$ and solves to give $\lambda=...$ | M1 |
| | $\mathbf{k}:\ p-3\lambda=-2-5\mu\Rightarrow$<br>$p-3(4)=-2-5(-1)\Rightarrow\underline{p=15}$<br><br>or $\mathbf{k}:\ p-3\lambda=3\Rightarrow$<br>$p-3(4)=3\Rightarrow\underline{p=15}$ | Equates **k** components, substitutes their $\lambda$ and their $\mu$ and solves to give $p=...$ **or** equates **k** components to give their "$p-3\lambda=$ the **k** value of $A$ found in part (a)", substitutes their $\lambda$ and solves to give $p=...$ | M1 |
| | | $p=15$ | A1 |
| | | | **[3]** |

| | | | |
|---|---|---|---|
| (c) | $\mathbf{d}_1 = \begin{pmatrix}0\\1\\-3\end{pmatrix},\ \mathbf{d}_2 = \begin{pmatrix}3\\4\\-5\end{pmatrix} \Rightarrow \begin{pmatrix}0\\1\\-3\end{pmatrix} \bullet \begin{pmatrix}3\\4\\-5\end{pmatrix}$ | Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. | M1 |
| | $\cos\theta = \pm K\left(\dfrac{0(3)+(1)(4)+(-3)(-5)}{\sqrt{(0)^2+(1)^2+(-3)^2}\,.\sqrt{(3)^2+(4)^2+(-5)^2}}\right)$ | An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. | dM1 (A1 on ePEN) |
| | $\cos\theta = \dfrac{19}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp) | anything that rounds to 31.82 | A1 |
| | | | [3] |
| (d) | $\overrightarrow{OB} = \begin{pmatrix}11\\9\\-7\end{pmatrix};\ \overrightarrow{AB} = \begin{pmatrix}11\\9\\-7\end{pmatrix} - \begin{pmatrix}5\\1\\3\end{pmatrix} = \begin{pmatrix}6\\8\\-10\end{pmatrix}$ or $\overrightarrow{AB} = 2\begin{pmatrix}3\\4\\-5\end{pmatrix} = \begin{pmatrix}6\\8\\-10\end{pmatrix}$<br>$\lvert\overrightarrow{AB}\rvert = \sqrt{6^2+8^2+(-10)^2}\ \{=10\sqrt{2}\}$ | **See notes** | M1 |
| | $\dfrac{d}{10\sqrt{2}} = \sin\theta$ | Writes down a correct trigonometric equation involving the shortest distance, $d$. Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$, oe. | dM1 |
| | $\{d = 10\sqrt{2}\sin 31.82... \Rightarrow\}\ d = 7.456540753... = 7.46$ (3sf) | anything that rounds to 7.46 | A1 |
| | | | [3]<br>11 |

| | | | |
|---|---|---|---|
| **4.** (b) | **Alternative method for part (b)**<br>$\left\{\begin{array}{l}3\times\mathbf{j}:\ -9+3\lambda = 15+12\mu\\ \quad\mathbf{k}:\ \ p-3\lambda = -2+5\mu\end{array}\right\}\ \ p-9 = 13+7\mu$ | Eliminates $\lambda$ to write down an equation in $p$ and $\mu$ | M1 |
| | $p-9 = 13+7(-1) \Rightarrow \underline{p=15}$ | Substitutes their $\mu$ and solves to give $p = ...$ | M1 |
| | | $p = 15$ | A1 |
| **4.** (d) | **Alternative Methods for part (d)** Let $X$ be the foot of the perpendicular from $B$ onto $l_1$<br>$\mathbf{d}_1 = \begin{pmatrix}0\\1\\-3\end{pmatrix},\ \overrightarrow{OX} = \begin{pmatrix}5\\-3\\15\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\-3\end{pmatrix} = \begin{pmatrix}5\\-3+\lambda\\15-3\lambda\end{pmatrix}$<br>$\overrightarrow{BX} = \begin{pmatrix}5\\-3+\lambda\\15-3\lambda\end{pmatrix} - \begin{pmatrix}11\\9\\-7\end{pmatrix} = \begin{pmatrix}-6\\-12+\lambda\\22-3\lambda\end{pmatrix}$<br>**Method 1** | | |
| | $\overrightarrow{BX}\bullet\mathbf{d}_1 = 0 \Rightarrow \begin{pmatrix}-6\\-12+\lambda\\22-3\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-3\end{pmatrix} = -12+\lambda-66+9\lambda = 0$<br>leading to $10\lambda - 78 = 0 \Rightarrow \lambda = \dfrac{39}{5}$ | (Allow a sign slip in copying $\mathbf{d}_1$)<br>Applies $\overrightarrow{BX}\bullet\mathbf{d}_1 = 0$ and solves the resulting equation to find a value for $\lambda$. | M1 |

| | | | |
|---|---|---|---|
| | $\overrightarrow{BX} = \begin{pmatrix} -6 \\ -12+\frac{39}{5} \\ 22-3\left(\frac{39}{5}\right) \end{pmatrix} = \begin{pmatrix} -6 \\ -\frac{21}{5} \\ -\frac{7}{5} \end{pmatrix}$ | Substitutes their value of $\lambda$ into their $\overrightarrow{BX}$. **Note:** This mark is dependent upon the previous M1 mark . | dM1 |
| | $d = BX = \sqrt{(-6)^2 + \left(-\frac{21}{5}\right)^2 + \left(-\frac{7}{5}\right)^2} = 7.456540753...$ | awrt 7.46 | A1 |
| | **Method 2** | | |
| | Let $\beta = \lvert\overrightarrow{BX}\rvert^2 = 36 + 144 - 24\lambda + \lambda^2 + 484 - 132\lambda + 9\lambda^2$ $= 10\lambda^2 - 156\lambda + 664$ So $\frac{d\beta}{d\lambda} = 20\lambda - 156 = 0 \Rightarrow \lambda = \frac{39}{5}$ | Finds $\beta = \lvert\overrightarrow{BX}\rvert^2$ in terms of $\lambda$, finds $\frac{d\beta}{d\lambda}$ and sets this result equal to 0 and finds a value for $\lambda$. | M1 |
| | $\lvert\overrightarrow{BX}\rvert^2 = 10\left(\frac{39}{5}\right)^2 - 156\left(\frac{39}{5}\right) + 664 = \frac{278}{5}$ | Substitutes their value of $\lambda$ into their $\lvert\overrightarrow{BX}\rvert^2$. **Note:** This mark is dependent upon the previous M1 mark . | dM1 |
| | $d = BX = \sqrt{\frac{278}{5}} = 7.456540753...$ | awrt 7.46 | A1 |

| | | **Question 4 Notes** |
|---|---|---|
| **4.** (a) | **M1** | Finds $\mu$ and substitutes their $\mu$ into $l_2$ |
| | **A1** | Point of intersection of $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$. Allow $\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ or $(5, 1, 3)$. |
| | **Note** | You cannot recover the answer for part (a) in part (c) or part (d). |
| (b) | **M1** | Equates **j** components, substitutes their $\mu$ and solves to give $\lambda = ...$ |
| | **M1** | Equates **k** components, substitutes their $\lambda$ and their $\mu$ and solves to give $p = ...$ **or** equates **k** components to give their "$p - 3\lambda$ = the **k** value of $A$" found in part (b). |
| | **A1** | $p = 15$ |
| (c) | **NOTE** | **Part (c) appears as M1A1A1 on ePEN, but now is marked as M1M1A1.** |
| | **M1** | Realisation that the dot product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. |
| | **Note** | Allow one slip in candidates copying down their direction vectors, $\mathbf{d}_1$ and $\mathbf{d}_2$. |
| | **dM1** | **dependent on the FIRST method mark being awarded.** An attempt to apply the dot product formula between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. |
| | **A1** | anything that rounds to 31.82. This can also be achieved by $180 - 148.1796... =$ awrt 31.82 |
| | **Note** | $\theta = 0.5553...^c$ is A0. |
| | **Note** | **M1A1 for** $\cos\theta = \left(\frac{0 - 16 - 60}{\sqrt{(0)^2 + (4)^2 + (-12)^2}\,.\sqrt{(-3)^2 + (-4)^2 + (5)^2}}\right) = \frac{-76}{\sqrt{160}.\sqrt{50}}$ |

| | | | |
|---|---|---|---|
| | ***Alternative Method: Vector Cross Product*** <br> **Only apply this scheme if it is clear that a candidate is applying a vector cross product method.** | | |
| | $\mathbf{d}_1 \times \mathbf{d}_2 = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -3 \\ 3 & 4 & -5 \end{vmatrix} = 7\mathbf{i} - 9\mathbf{j} - 3\mathbf{k} \right\}$ | Realisation that the vector cross product is required between $\pm A\mathbf{d}_1$ and $\pm B\mathbf{d}_2$. | M1 |
| | $\sin\theta = \dfrac{\sqrt{(7)^2 + (-9)^2 + (3)^2}}{\sqrt{(0)^2 + (1)^2 + (-3)^2}\,.\sqrt{(3)^2 + (4)^2 + (-5)^2}}$ | An attempt to apply the vector cross product formula | dM1 (A1 on ePEN) |
| | $\sin\theta = \dfrac{\sqrt{139}}{\sqrt{10}.\sqrt{50}} \Rightarrow \theta = 31.8203116... = 31.82$ (2 dp) | anything that rounds to 31.82 | A1 |

| | | |
|---|---|---|
| (d) | M1 | Full method for finding $B$ **and** for finding the magnitude of $\overrightarrow{AB}$ or the magnitude of $\overrightarrow{BA}$. |
| | dM1 | **dependent on the first method mark being awarded.** <br> Writes down correct trigonometric equation involving the shortest distance, $d$. <br> Eg: $\dfrac{d}{\text{their } AB} = \sin\theta$ **or** $\dfrac{d}{\text{their } AB} = \cos(90-\theta)$, o.e., where " their $AB$" is a value. <br> and $\theta =$ "their $\theta$" or stated as $\theta$ |
| | A1 | anything that rounds to 7.46 |

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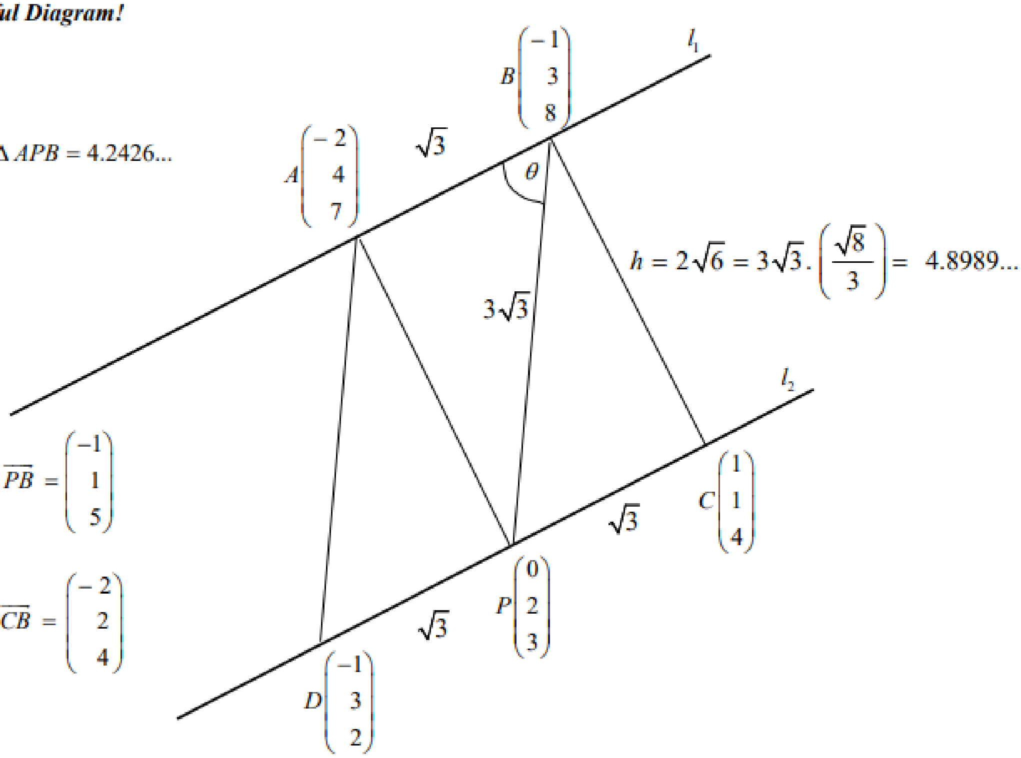
4.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** | $\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ | | |
| (a) | $\overrightarrow{AB} = = \pm((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}));\ = \mathbf{i} - \mathbf{j} + \mathbf{k}$ | | M1; A1 |
| | | | **[2]** |
| (b) | $\{l_1 : \mathbf{r}\} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ | | B1ft |
| | | | **[1]** |
| (c) | $\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$ | | M1 |
| | $\{\cos\theta =\}\ \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{PB}\rvert} = \dfrac{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2}\,.\sqrt{(-1)^2 + (1)^2 + (5)^2}}$ | Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{PB}$ or $\overrightarrow{BP})$. | M1 |
| | $\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$ | Correct proof | A1 cso |
| | | | **[3]** |

| | | | | |
|---|---|---|---|---|
| (d) | $\{l_2 : \mathbf{r} =\}\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ | $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$, $\mathbf{p} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d}$ = their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$. | M1 | |
| | | Correct vector equation. | A1 ft | |
| | | | | [2] |
| (e) | $\overrightarrow{OC} = \begin{pmatrix}0\\2\\3\end{pmatrix} + \begin{pmatrix}1\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix}0\\2\\3\end{pmatrix} - \begin{pmatrix}1\\-1\\1\end{pmatrix}$ | Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ – their $\overrightarrow{AB}$ | M1 | |
| | | At least one set of coordinates are correct. | A1 ft | |
| | $\{C(1, 1, 4),\ D(-1, 3, 2)\}$ | Both sets of coordinates are correct. | A1 ft | |
| | | | | [3] |
| (f) **Way 1** | $\dfrac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin\theta$ | $\dfrac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$ | M1 | |
| | $h = \sqrt{27}\sin(70.5...)\left\{= \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9\right\}$ | $\sqrt{27}\sin(70.5...)$ or $\sqrt{27}.\dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent | A1 oe | |
| | Area $ABCD = \dfrac{1}{2}2\sqrt{6}\left(\sqrt{3} + 2\sqrt{3}\right)$ | $\dfrac{1}{2}(\text{their } h)(\text{their } AB + \text{their } CD)$ | dM1 | |
| | $\left\{= \dfrac{1}{2}2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18}\right\} = \underline{9\sqrt{2}}$ | $9\sqrt{2}$ | A1 **cao** | |
| | | | | [4] 15 |

**8. (f)** ***Helpful Diagram!***

Area $\Delta APB$ = 4.2426...




$\overrightarrow{DA} = \overrightarrow{PB} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$

$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$

| | | | |
|---|---|---|---|
| | $\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$, so $BC \perp AB$ | Candidates do not need to prove this result for part (f) | |

| | | | |
|---|---|---|---|
| **8. (f) Way 2** | $h = \lvert\overrightarrow{CB}\rvert = \sqrt{(-2)^2+(2)^2+(4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989...$ | Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$ | M1 |
| | | $\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$ | A1 oe |
| | Area $ABCD = \frac{1}{2}\sqrt{24}\left(\sqrt{3}+2\sqrt{3}\right)$ or $\frac{1}{2}\sqrt{24}\sqrt{3}+\sqrt{24}\sqrt{3}$ | $\frac{1}{2}h(\text{their } AB + \text{their } CD)$ | dM1 oe |
| | $= \underline{9\sqrt{2}}$ | $9\sqrt{2}$ | A1 **cso** |
| | | | **[4]** |
| **Way3** | **<u>Finds the area of either triangle *APB* or *APD* or *BCP* and triples the result.</u>** | | |
| **8. (f)** | Area $\Delta APB = \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\theta$ | Attempts $\frac{1}{2}$(their $AB$)(their $PB$)$\sin\theta$ | M1 |
| | $= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ | $\frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ or $3\sqrt{2}$ or awrt 4.24 or equivalent | A1 |
| | Area $ABCD = 3\left(3\sqrt{2}\right)$ | $3 \times$ Area of $\Delta APB$ | dM1 |
| | $= \underline{9\sqrt{2}}$ | $9\sqrt{2}$ | A1 **cso** |
| | | | **[4]** |

| | | **Question 8 Notes** |
|---|---|---|
| **8.** (a) | **M1** | Finding the difference (either way) between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. |
| | **A1** | $\mathbf{i}-\mathbf{j}+\mathbf{k}$ or $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $(1, -1, 1)$ or benefit of the doubt $\begin{matrix}1\\-1\\1\end{matrix}$ |
| (b) | **B1ft** | $\{\mathbf{r}\} = \begin{pmatrix}-2\\4\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ **or** $\{\mathbf{r}\} = \begin{pmatrix}-1\\3\\8\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$, with $\overrightarrow{AB}$ or $\overrightarrow{BA}$ correctly followed through from (a). |
| | **Note** | $\mathbf{r} =$ is not needed. |
| (c) | **M1** | An attempt to find either the vector $\overrightarrow{PB}$ or $\overrightarrow{BP}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. |
| | **M1** | Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB}\text{ or }\overrightarrow{BP}\right)$. |
| | **A1** | Obtains $\{\cos\theta\} = \frac{1}{3}$ ***by correct solution only.*** |
| | **Note** | If candidate starts by applying $\frac{\overrightarrow{AB}\bullet\overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{PB}\rvert}$ correctly (without reference to $\cos\theta = ...$) they can gain both 2nd M1 and A1 mark. |
| | **Note** | Award the final A1 mark if candidate achieves $\{\cos\theta\} = \frac{1}{3}$ by either taking the dot product between (i) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ **or** (ii) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly. |

| | | |
|---|---|---|
| | **Note** | Award final A0, cso for those candidates who take the dot product between (iii) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ or (iv) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$<br>They will usually find $\{\cos\theta\} = -\frac{1}{3}$ or may fudge $\{\cos\theta\} = \frac{1}{3}$.<br>If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso |

| | | | |
|---|---|---|---|
| (c) | **Alternative Method 1: The Cosine Rule** | | |
| | $\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$ | Mark in the same way as the main scheme. | M1 |
| | Note $\lvert\overrightarrow{PB}\rvert = \sqrt{27}$, $\lvert\overrightarrow{AB}\rvert = \sqrt{3}$ and $\lvert\overrightarrow{PA}\rvert = \sqrt{24}$ | | |
| | $(\sqrt{24})^2 = (\sqrt{27})^2 + (\sqrt{3})^2 - 2(\sqrt{27})(\sqrt{3})\cos\theta$ | Applies the cosine rule the correct way round | M1 oe |
| | $\cos\theta = \frac{27+3-24}{18} = \frac{1}{3}$ | Correct proof | A1 cso |
| | | | **[3]** |

| | | | |
|---|---|---|---|
| **8.** (c) | **Alternative Method 2: Right-Angled Trigonometry** | | |
| | $\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$ | Mark in the same way as the main scheme. | M1 |
| | Either $(\sqrt{24})^2 + (\sqrt{3})^2 = (\sqrt{27})^2$<br>or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-2\\2\\4\end{pmatrix} = -2-2+4=0$ | Confirms $\Delta PAB$ is right-angled | M1 |
| | So, $\left\{\cos\theta = \frac{AB}{PB} \Rightarrow\right\} \cos\theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}$ | Correct proof | A1 cso |
| | | | **[3]** |

| | | |
|---|---|---|
| (d) | **M1** | Writing down a line in the form $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix}0\\2\\3\end{pmatrix}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$ $\mathbf{d} =$ their $\overrightarrow{AB}$,<br>or a multiple of their $\overrightarrow{AB}$ found in part (a). |
| | **A1ft** | Writing $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d} =$ their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a). |
| | **Note** | $\mathbf{r} =$ is not needed. |
| | **Note** | Using the same scalar parameter as in part (b) is fine for A1. |
| (e) | **M1** | Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$. |
| | **Note** | This can be implied at least two out of three correct components for either their $C$ or their $D$. |
| | **A1ft** | At least one set of coordinates are correct. Ignore labelling of $C$, $D$ |
| | **A1ft** | Both sets of coordinates are correct. Ignore labelling of $C$, $D$ |
| | **Note** | You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a). |

| | | |
|---|---|---|
| (f) | M1 | Way 1: $\frac{h}{\text{their }\lvert\overline{PB}\rvert} = \sin\theta$<br>Way 2: Attempts $\lvert\overline{PA}\rvert$ or $\lvert\overline{CB}\rvert$<br>Way 3: Attempts $\frac{1}{2}$(their $PB$)(their $AB$)$\sin\theta$ |
| | Note | Finding $AD$ by itself is M0. |
| | A1 | Either<br>• $h = \sqrt{27}\sin(70.5...)$ or $\lvert\overline{PA}\rvert = \lvert\overline{CB}\rvert = \sqrt{24}$ or equivalent. (See Way 1 and Way 2)<br>or<br>• the area of either triangle $APB$ or $APD$ or $BDP$ $= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\left(70.5...\right)$ o.e. (See Way 3). |
| | dM1 | **which is dependent on the 1st M1 mark.**<br>A full method to find the area of trapezium $ABCD$. (See Way 1, Way 2 and Way 3). |
| | A1 | $9\sqrt{2}$ from a correct solution only. |
| | Note | A decimal answer of 12.7279... (without a correct **exact** answer) is A0. |

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5.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** | $l: \mathbf{r} = \begin{pmatrix}13\\8\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\2\\-1\end{pmatrix}$, $A(3,-2,6)$, $\overrightarrow{OP} = \begin{pmatrix}-p\\0\\2p\end{pmatrix}$ | | |
| **(a)** | $\{\overrightarrow{PA}\} = \begin{pmatrix}3\\-2\\6\end{pmatrix} - \begin{pmatrix}-p\\0\\2p\end{pmatrix}$ \| $\{\overrightarrow{AP}\} = \begin{pmatrix}-p\\0\\2p\end{pmatrix} - \begin{pmatrix}3\\-2\\6\end{pmatrix}$ | Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OP}$. Ignore labelling. | M1 |
| | $= \begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix}$ \| $= \begin{pmatrix}-3-p\\2\\2p-6\end{pmatrix}$ | Correct difference. | A1 |
| | $\begin{pmatrix}3+p\\-2\\6-2p\end{pmatrix} \bullet \begin{pmatrix}2\\2\\-1\end{pmatrix} = 6+2p-4-6+2p=0$ | See notes. | M1 |
| | $p=1$ | | A1 cso |
| | | | **[4]** |

| | | | |
|---|---|---|---|
| **(b)** | $\lvert AP \rvert = \sqrt{4^2 + (-2)^2 + 4^2}$ or $\lvert AP \rvert = \sqrt{(-4)^2 + 2^2 + (-4)^2}$ | See notes. | M1 |
| | So, $PA$ or $AP = \sqrt{36}$ or $6$ **cao** | | A1 **cao** |
| | It follows that, $AB = \text{"}6\text{"}$ $\{= PA\}$ or $PB = \text{"}6\sqrt{2}\text{"}$ $\{= \sqrt{2}\,PA\}$ | See notes. | B1 ft |
| | {Note that $AB = \text{"}6\text{"} = 2$(the modulus of the direction vector of $l$)} <br> $\overrightarrow{OB} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} \pm 2\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ or <br> $\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - 7\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ | Uses a correct method in order to find both possible sets of coordinates of $B$. | M1 |
| | $= \begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix}$ | Both coordinates are correct. | A1 **cao** |
| | | | **[5]** <br> **9** |

| **Notes for Question 8** | |
|---|---|
| **8. (a)** | **M1:** Finds the difference between $\overline{OA}$ and $\overline{OP}$. Ignore labelling. <br> If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference. <br> **A1:** Accept any of $\begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix}$ or $(3+p)\mathbf{i} - 2\mathbf{j} + (6-2p)\mathbf{k}$ or $\begin{pmatrix} -3-p \\ 2 \\ 2p-6 \end{pmatrix}$ or $(-3-p)\mathbf{i} + 2\mathbf{j} + (2p-6)\mathbf{k}$ |

| **Notes for Question 8 Continued** | |
|---|---|
| **8. (a)** | **M1:** ***Applies*** the formula $\overrightarrow{PA} \bullet \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{AP} \bullet \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ correctly to give a linear equation in $p$ which is set equal to zero. **Note:** The dot product can also be with $\pm k\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$. Eg: Some candidates may find <br> $\begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \\ -5 \end{pmatrix}$, for instance, and use this in their dot product which is fine for M1. <br> **A1:** Finds $p = 1$ from ***a correct solution only.*** <br> **Note:** The direction of subtraction is not important in part (a). |
| **(b)** | **M1:** Uses their value of $p$ and Pythagoras to obtain a numerical expression for either $AP$ or $PA$ or $AP^2$ or $PA^2$. Eg: $PA$ or $AP = \sqrt{4^2 + (-2)^2 + 4^2}$ or $\sqrt{(-4)^2 + 2^2 + (-4)^2}$ or $\sqrt{4^2 + 2^2 + 4^2}$ <br> or $PA^2$ or $AP^2 = 4^2 + (-2)^2 + 4^2$ or $(-4)^2 + 2^2 + (-4)^2$ or $4^2 + 2^2 + 4^2$ <br> **A1:** $AP$ or $PA = \sqrt{36}$ or $6$ **cao** or $AP^2 = 36$ **cao** <br> **B1ft:** States or it is clear from their working that $AB = \text{"}6\text{"}$ $\{=$ their evaluated $PA\}$ or <br> $PB = \text{"}6\text{"}\sqrt{2}$ $\{= \sqrt{2}$ (their evaluated $PA$)$\}$. |
| | **Note:** So a correct follow length is required here for either $AB$ or $PB$ using their evaluated $PA$. <br> **Note:** This mark may be found on a diagram. <br> **Note:** If a candidate states that $\lvert \overrightarrow{AP} \rvert = \lvert \overrightarrow{AB} \rvert$ and then goes on to find $\lvert \overrightarrow{AP} \rvert = 6$ then the B1 mark can be implied. <br> **IMPORTANT:** This mark may be implied as part of expressions such as: <br> $\{AB =\} \sqrt{(10+2\lambda)^2 + (10+2\lambda)^2 + (-5-\lambda)^2} = \mathbf{6}$ or $\{AB^2 =\}$ $(10+2\lambda)^2 + (10+2\lambda)^2 + (-5-\lambda)^2 = \mathbf{36}$ <br> or $\{PB =\} \sqrt{(14+2\lambda)^2 + (8+2\lambda)^2 + (-1-\lambda)^2} = \mathbf{6\sqrt{2}}$ or $\{PB^2 =\}$ $(14+2\lambda)^2 + (8+2\lambda)^2 + (-1-\lambda)^2 = \mathbf{72}$ |

**M1:** Uses a full method in order to find **both** possible sets of coordinates of $B$:

Eg 1: $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} \pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ Eg 2: $\overrightarrow{OB} = \begin{pmatrix}13\\8\\1\end{pmatrix} - 3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}13\\8\\1\end{pmatrix} - 7\begin{pmatrix}2\\2\\-1\end{pmatrix}$

**Note:** If a candidate achieves at least one of the correct $(7, 2, 4)$ or $(-1, -6, 8)$ then award SC M1 here.

**Note:** $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} - 3\begin{pmatrix}2\\2\\-1\end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} - 7\begin{pmatrix}2\\2\\-1\end{pmatrix}$ is M0.

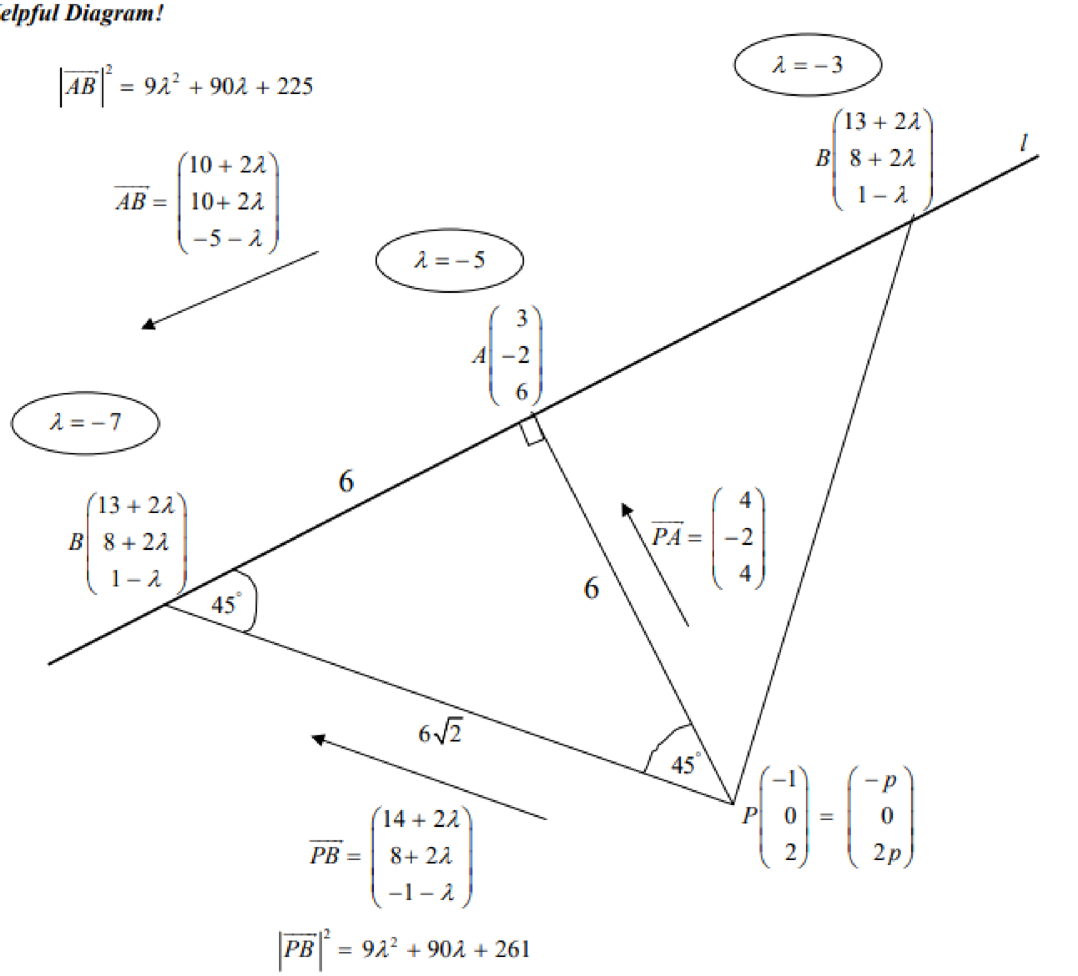
**A1:** For both $(7, 2, 4)$ and $(-1, -6, 8)$. Accept vector notation or $\mathbf{i}, \mathbf{j}, \mathbf{k}$ notation.

**Note:** All the marks are accessible in part (b) if $p = 1$ is found from incorrect working in part (a).

**Note: Imply M1A1B1 and award M1** for candidates who write: $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} \pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$, with little or no earlier working.

**Notes for Question 8 Continued**

**8.** ***Helpful Diagram!***



| | | |
|---|---|---|
| **8. (b)** | **Way 2:** Setting $AB = "6"$ or $AB^2 = "36"$ **Note:** It is possible for you to apply the main scheme for Way 2.<br>$\{AB = "6" \Rightarrow AB^2 = "36" \Rightarrow\}$ $(10+2\lambda)^2 + (10+2\lambda)^2 + (-5-\lambda)^2 = "36"$ **B1ft** could be implied here.<br>$9\lambda^2 + 90\lambda + 225 = 36 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$<br>$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$<br>$\lambda = -3, -7$<br>Then apply final M1 A1 as in the original scheme. | ... M1 A1 |
| **8. (b)** | **Way 3:** Setting $PB = "6\sqrt{2}"$ or $PB^2 = "72"$ **Note:** It is possible for you to apply the main scheme for Way 3.<br>$\{PB = "6"\sqrt{2} \Rightarrow PB^2 = "72" \Rightarrow\}$ $(14+2\lambda)^2 + (8+2\lambda)^2 + (-1-\lambda)^2 = "72"$ **B1ft** could be implied here.<br>$9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$<br>$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$<br>$\lambda = -3, -7$<br>Then apply final M1 A1 as in the original scheme. | ... M1 A1 |

**Notes for Question 8 Continued**

| | | |
|---|---|---|
| **8. (b)** | **(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).**<br>**Way 4:** Using the dot product formula between $\overrightarrow{PA}$ and $\overrightarrow{PB}$, ie: $\cos 45^\circ = \dfrac{\overrightarrow{PA} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{PA}\rvert . \lvert\overrightarrow{PB}\rvert}$.<br>$\overrightarrow{PA} \bullet \overrightarrow{PB} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 14+2\lambda \\ 8+2\lambda \\ -1-\lambda \end{pmatrix} = 56 + 8\lambda - 16 - 4\lambda - 4 - 4\lambda = 36$<br>$\{\cos 45^\circ =\}\ \dfrac{1}{\sqrt{2}} = \dfrac{36}{6\sqrt{9\lambda^2 + 90\lambda + 261}}$ — For finding $\lvert\overrightarrow{PA}\rvert$ as before. $\sqrt{36}$ or $6$. $\lvert\overrightarrow{PB}\rvert = \sqrt{9\lambda^2 + 90\lambda + 261}$ | M1<br>A1 **cao**<br>B1 oe |
| | $\dfrac{1}{2} = \dfrac{36}{9\lambda^2 + 90\lambda + 261}$<br>$9\lambda^2 + 90\lambda + 261 = 72 \Rightarrow 9\lambda^2 + 90\lambda + 189 = 0$<br>$\lambda^2 + 10\lambda + 21 = 0 \Rightarrow (\lambda+3)(\lambda+7) = 0$<br>$\lambda = -3, -7$<br>Then apply final M1 A1 as in the original scheme. | ... M1 A1 |

<table>
<tr><td>8. (b)</td><td colspan="2">(You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 5).<br><br>Way 5: Using the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$, ie: $\cos 45^{\circ} = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{|\overrightarrow{AB}|.|\overrightarrow{PB}|}$</td></tr>
<tr><td></td><td>$\cos 45^{\circ} = \dfrac{1}{\sqrt{2}} = \dfrac{\begin{pmatrix}10+2\lambda\\10+2\lambda\\-5-\lambda\end{pmatrix} \bullet \begin{pmatrix}14+2\lambda\\8+2\lambda\\-1-\lambda\end{pmatrix}}{\sqrt{9\lambda^2+90\lambda+225}\ \sqrt{9\lambda^2+90\lambda+261}}$<br><br>Attempts the dot product formula between $\overrightarrow{AB}$ and $\overrightarrow{PB}$.<br>Correct statement with $|\overrightarrow{AB}|$ and $|\overrightarrow{PB}|$ simplified as shown.<br>Either $|\overrightarrow{AB}| = \sqrt{9\lambda^2+90\lambda+225}$ or $|\overrightarrow{PB}| = \sqrt{9\lambda^2+90\lambda+261}$</td><td>M1<br><br>A1<br><br>B1</td></tr>
<tr><td></td><td>$\{\cos 45^{\circ} =\}\ \dfrac{1}{\sqrt{2}} = \dfrac{140+20\lambda+28\lambda+4\lambda^2+80+20\lambda+16\lambda+4\lambda^2+5+5\lambda+\lambda+\lambda^2}{\sqrt{9\lambda^2+90\lambda+225}\ \sqrt{9\lambda^2+90\lambda+261}}$<br><br>$\{\cos 45^{\circ} =\}\ \dfrac{1}{\sqrt{2}} = \dfrac{9\lambda^2+90\lambda+225}{\sqrt{9\lambda^2+90\lambda+225}\ \sqrt{9\lambda^2+90\lambda+261}}$<br><br>$\dfrac{1}{2} = \dfrac{(9\lambda^2+90\lambda+225)^2}{(9\lambda^2+90\lambda+225)(9\lambda^2+90\lambda+261)}$<br><br>$\dfrac{1}{2} = \dfrac{(9\lambda^2+90\lambda+225)}{(9\lambda^2+90\lambda+261)}$<br><br>$9\lambda^2+90\lambda+261 = 2(9\lambda^2+90\lambda+225) \Rightarrow 9\lambda^2+90\lambda+189=0$<br>$\lambda^2+10\lambda+21=0 \Rightarrow (\lambda+3)(\lambda+7)=0$<br>$\lambda = -3, -7$<br><br>Then apply final M1 A1 as in the original scheme.</td><td><br><br><br><br><br><br><br><br><br>... M1 A1</td></tr>
</table>

<table>
<tr><th colspan="3">Notes for Question 8 Continued</th></tr>
<tr><td>8. (b)</td><td>Way 6:<br>$\overrightarrow{PA} = \begin{pmatrix}4\\-2\\4\end{pmatrix} = 2\begin{pmatrix}2\\-1\\2\end{pmatrix}$ and direction vector of $l$ is $\mathbf{d} = \begin{pmatrix}2\\2\\-1\end{pmatrix}$<br><br>So, $|\overrightarrow{PA}| = 2\,|\mathbf{d}|$ or $PA = 2\,|\mathbf{d}|$<br>A correct statement relating these distances (and not vectors)<br><br>Apply final M1 A1 as in the original scheme.</td><td><br><br><br>M1 A1 B1<br><br>... M1 A1</td></tr>
<tr><td></td><td colspan="2">Note: $\overrightarrow{PA} = 2\mathbf{d}$ with no other creditable working is M0A0B0...<br><br>Note: $\overrightarrow{PA} = 2\mathbf{d}$, followed by $\overrightarrow{OB} = \begin{pmatrix}3\\-2\\6\end{pmatrix} \pm 2\begin{pmatrix}2\\2\\-1\end{pmatrix}$ is M1A1B1M1 and the final A1 mark is for both sets of correct coordinates.</td></tr>
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6.

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** | (a) $\overrightarrow{AB}=\begin{pmatrix}8\\3\\4\end{pmatrix}-\begin{pmatrix}10\\2\\3\end{pmatrix}=\begin{pmatrix}-2\\1\\1\end{pmatrix}$ | M1 A1 **(2)** |
| | (b) $\mathbf{r}=\begin{pmatrix}10\\2\\3\end{pmatrix}+t\begin{pmatrix}-2\\1\\1\end{pmatrix}$ $\qquad$ $\mathbf{r}=\begin{pmatrix}8\\3\\4\end{pmatrix}+t\begin{pmatrix}-2\\1\\1\end{pmatrix}$ | M1 A1ft **(2)** |
| | (c) $\overrightarrow{CP}=\begin{pmatrix}10-2t\\2+t\\3+t\end{pmatrix}-\begin{pmatrix}3\\12\\3\end{pmatrix}=\begin{pmatrix}7-2t\\t-10\\t\end{pmatrix}$ | M1 A1 |
| | $\begin{pmatrix}7-2t\\t-10\\t\end{pmatrix}.\begin{pmatrix}-2\\1\\1\end{pmatrix}=-14+4t+t-10+t=0$ | M1 |
| | Leading to $\quad t=4$ | A1 |
| | Position vector of $P$ is $\begin{pmatrix}10-8\\2+4\\3+4\end{pmatrix}=\begin{pmatrix}2\\6\\7\end{pmatrix}$ | M1 A1 **(6)** |
| | | **[10]** |
| | *Alternative working for* (c) | |
| | $\overrightarrow{CP}=\begin{pmatrix}8-2t\\3+t\\4+t\end{pmatrix}-\begin{pmatrix}3\\12\\3\end{pmatrix}=\begin{pmatrix}5-2t\\t-9\\t+1\end{pmatrix}$ | M1 A1 |
| | $\begin{pmatrix}5-2t\\t-9\\t+1\end{pmatrix}.\begin{pmatrix}-2\\1\\1\end{pmatrix}=-10+4t+t-9+t+1=0$ | M1 |
| | Leading to $\quad t=3$ | A1 |
| | Position vector of $P$ is $\begin{pmatrix}8-6\\3+3\\4+3\end{pmatrix}=\begin{pmatrix}2\\6\\7\end{pmatrix}$ | M1 A1 **(6)** |

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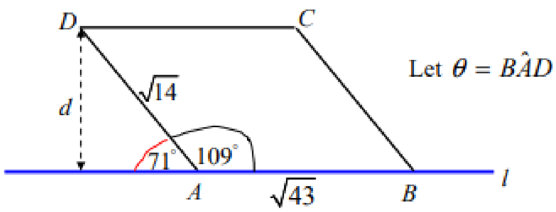
7.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.** | $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ , $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ | | |
| (a) | $\overrightarrow{AB} = = \pm((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ | | M1; A1 **[2]** |
| (b) | $l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ | See notes | M1 A1ft **[2]** |
| |  Let $\theta = B\hat{A}D$ | Let $d$ be the shortest distance from $C$ to $l$. | |
| (c) | $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ | | M1 |
| | $\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$ | Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$. | M1 |
| | $\cos\theta = \pm\left(\dfrac{-9+6-5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$ | Correct followed through expression or **equation**. | A1 $\sqrt{}$ |
| | $\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109$ (nearest °) | awrt 109 | A1 **cso** **AG** **[4]** |
| (d) | $\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$<br>$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ | | M1 |
| | So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$ | | A1 **[2]** |
| (e) | Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$ | awrt 23.2 | M1; dM1 A1 **[3]** |
| (f) | $\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$ | | M1 |
| | $\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$ | awrt 3.54 | A1 **[2]** **15** |

| | |
|---|---|
| 7. (a) | **M1:** Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.<br>Can be implied by two out of three components correct in $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ or $-3\mathbf{i}-3\mathbf{j}-5\mathbf{k}$<br>**A1:** $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ |
| (b) | **M1:** An expression of the form $(3\text{ component vector})\pm\lambda(3\text{ component vector})$<br>**A1ft:** $\mathbf{r}=\overrightarrow{OA}+\lambda(\text{their }\pm\overrightarrow{AB})$ or $\mathbf{r}=\overrightarrow{OB}+\lambda(\text{their }\pm\overrightarrow{AB})$.<br>**Note:** Candidate must begin writing their line as $\mathbf{r}=$ or $l=\ldots$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=\ldots$ So, Line = ... would be A0. |
| (c) | **M1:** An attempt to find either the vector $\overrightarrow{AD}$ or $\overrightarrow{DA}$.<br>Can be implied by two out of three components correct in $-3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ or $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, respectively.<br>**M1:** Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$.<br>**A1ft:** Correct followed through expression or **equation**. The dot product must be correctly followed through correctly and the square roots although they can be un-simplified must be followed through correctly.<br>**A1:** Obtains an angle of awrt 109 ***by correct solution only***.<br>Award the final A1 mark if candidate achieves awrt 109 by either taking the dot product between:<br>(i) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (ii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.<br>Award A0, cso for those candidates who take the dot product between:<br>(iii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$.<br>They will usually find awrt 71 and apply 180 – awrt 71 to give awrt 109. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso. If still in doubt, here, send to review. |
| (d) | **M1:** Applies either $\overrightarrow{OD}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OB}$ + their $\overrightarrow{AD}$.<br>This mark can be implied by two out of three correctly followed through components in their $\overrightarrow{OD}$.<br>**A1:** For $2\mathbf{i}+4\mathbf{j}+9\mathbf{k}$. |
| (e) | **M1:** $\frac{1}{2}(\text{their }AB)(\text{their }CB)\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$. Awrt 11.6 will usually imply this mark.<br>**dM1:** Multiplies this by 2 for the parallelogram. Can be implied.<br>**Note:** $\frac{1}{2}((\text{their }AB+\text{their }AB))(\text{their }CB)\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$<br>**A1:** awrt 23.2 |
| (f) | **M1:** $\dfrac{d}{\text{their }AD}=\sin(\text{their }109^\circ\text{ or }71^\circ\text{ from (b)})$ or $(\text{their }AB)\ d=(\text{their Area }ABCD)$<br>Award M0 for $(\text{their }AB)$ in part (f), if the area of their parallelogram in part (e) is $(\text{their }AB)(\text{their }CB)$.<br>Award M0 for $\dfrac{d}{\text{their }\sqrt{43}}=\sin 71$ or $(\text{their }\sqrt{14})d=23.19894905\ldots$<br>**A1:** awrt 3.54<br>**Note: Some candidates will use their answer to part (f) in order to answer part (e).** |

| | | |
|---|---|---|
| **7.** | *Alternative method for part (c): Applying the cosine rule:* | |
| | $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}-1\\1\\4\end{pmatrix} - \begin{pmatrix}2\\-1\\5\end{pmatrix} = \begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$ | **M1:** as above. |
| | $\overrightarrow{DB} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}5\\2\\10\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}6\\1\\6\end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix}-6\\-1\\-6\end{pmatrix}$ | |
| | So $\lvert\overrightarrow{AB}\rvert = \sqrt{43}$, $\lvert\overrightarrow{AD}\rvert = \sqrt{14}$ and $\lvert\overrightarrow{DB}\rvert = \sqrt{73}$ | |
| | $\cos\theta = \dfrac{(\sqrt{43})^2 + (\sqrt{14})^2 - (\sqrt{73})^2}{2\sqrt{43}.\sqrt{14}}$ | **M1:** Cosine rule structure of $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$ assigned each of $\lvert\overrightarrow{AB}\rvert, \lvert\overrightarrow{AD}\rvert$ and $\lvert\overrightarrow{DB}\rvert$ in any order as their *a*, *b* and *c*.<br>**A1:** Correct application of cosine rule. |
| | $\left\{\cos\theta = \dfrac{-16}{2\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544...\right\} = 109$ (nearest °) | A1: awrt 109 (no errors seen). **AG** |
| | *Alternative method for part (d):* | |
| | $\overrightarrow{OE} = \begin{pmatrix}2\\-1\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\5\end{pmatrix}$ | |
| | $\overrightarrow{DE} = \begin{pmatrix}2+3\lambda\\-1+3\lambda\\5+5\lambda\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}3+3\lambda\\-2+3\lambda\\1+5\lambda\end{pmatrix}$ | |
| | $\overrightarrow{DE} \bullet \overrightarrow{AB} = 0 \Rightarrow \begin{pmatrix}3+3\lambda\\-2+3\lambda\\1+5\lambda\end{pmatrix} \bullet \begin{pmatrix}3\\3\\5\end{pmatrix} = 0$ | **M1:** Takes the dot product between $\overrightarrow{DE}$ and $\overrightarrow{AB}$ and progresses to find a value of $\lambda$ |
| | $9 + 9\lambda - 6 + 9\lambda + 5 + 3\lambda = 0 \Rightarrow \lambda = -\dfrac{8}{43}$ | |
| | $\overrightarrow{DE} = \begin{pmatrix}2+3\lambda\\-1+3\lambda\\5+5\lambda\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}\frac{103}{43}\\-\frac{110}{43}\\\frac{3}{43}\end{pmatrix}$ | **dM1:** Uses their value of $\lambda$ to find $\overrightarrow{DE}$ |
| | Length DE = 3.537806563... | **A1:** awrt 3.54 |

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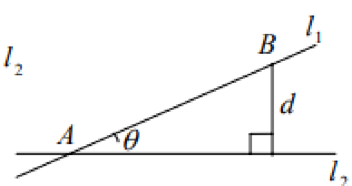
8.

| Question Number | Scheme | Marks | |
|---|---|---|---|
| **6.** | (a) **i**: $6-\lambda=-5+2\mu$ | | |
| | **j**: $-3+2\lambda=15-3\mu$ Any two equations | M1 | |
| | leading to $\lambda=3,\ \mu=4$ | M1 A1 | |
| | $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+3\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ *or* $\mathbf{r}=\begin{pmatrix}-5\\15\\3\end{pmatrix}+4\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}3\\3\\7\end{pmatrix}$ | M1 A1 | |
| | **k**: LHS $=-2+3(3)=7$, RHS $=3+4(1)=7$ | B1 | **(6)** |
| | (As LHS = RHS, lines intersect) | | |
| | *Alternatively* for B1, showing that $\lambda=3$ and $\mu=4$ both give $\begin{pmatrix}3\\3\\7\end{pmatrix}$ | | |
| | (b) $\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\-3\\1\end{pmatrix}=-2-6+3=\sqrt{14}\sqrt{14}\cos\theta$ $(\theta\approx110.92°)$ | M1 A1 | |
| | Acute angle is $69.1°$ awrt 69.1 | A1 | **(3)** |
| | (c) $\mathbf{r}=\begin{pmatrix}6\\-3\\-2\end{pmatrix}+1\begin{pmatrix}-1\\2\\3\end{pmatrix}=\begin{pmatrix}5\\-1\\1\end{pmatrix}$ $(\Rightarrow B$ lies on $l_1)$ | B1 | **(1)** |
| | (d) Let $d$ be shortest distance from $B$ to $l_2$   | | |
| | $\overrightarrow{AB}=\begin{pmatrix}5\\-1\\1\end{pmatrix}-\begin{pmatrix}3\\3\\7\end{pmatrix}=\begin{pmatrix}2\\-4\\-6\end{pmatrix}$ | M1 | |
| | $\lvert\overrightarrow{AB}\rvert=\sqrt{\left(2^2+(-4)^2+(-6)^2\right)}=\sqrt{56}$ awrt 7.5 | A1 | |
| | $\frac{d}{\sqrt{56}}=\sin\theta$ | M1 | |
| | $d=\sqrt{56}\sin 69.1°\approx 6.99$ awrt 6.99 | A1 | **(4)** |
| | | | **[14]** |

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9.

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** (a) | $\overrightarrow{AB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ | M1 A1 (2) |
| (b) | $\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$<br>or $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ | M1 A1ft (2) |
| (c) | $\overrightarrow{AC} = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$<br>$= \mathbf{i} + (p+3)\mathbf{j} - 6\mathbf{k}$ or $\overrightarrow{CA}$<br>$\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} . \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$<br>$-3 + 5p + 15 + 18 = 0$<br>Leading to $p = -6$ | B1<br><br>M1<br><br>M1 A1 (4) |
| (d) | $AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2 \quad (=46)$<br>$AC = \sqrt{46}$ accept awrt 6.8 | M1<br>A1<br>(2)<br>**[10]** |

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10.

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** | (a) **j** components $3+2\lambda=9 \Rightarrow \lambda=3$ $(\mu=1)$ | M1 A1 |
| | Leading to $C:(5,9,-1)$ accept vector forms | A1 **(3)** |
| | (b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$ | M1 |
| | $\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$ use of scalar product | M1 A1 |
| | $\angle ACB=57.95°$ awrt 57.95° | A1 **(4)** |
| | (c) $A:(2,3,-4)$ $B:(-5,9,-5)$ | |
| | $\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$ | |
| | $AC^2=3^2+6^2+3^2 \Rightarrow AC=3\sqrt{6}$ | M1 A1 |
| | $BC^2=10^2+4^2 \Rightarrow BC=2\sqrt{29}$ | A1 |
| | $\triangle ABC=\frac{1}{2}AC\times BC\sin\angle ACB$ | |
| | $=\frac{1}{2}3\sqrt{6}\times2\sqrt{29}\sin\angle ACB\approx33.5$ $15\sqrt{5}$, awrt 34 | M1 A1 **(5)** |
| | | **[12]** |
| | *Alternative method for* (b) *and* (c) | |
| | (b) $A:(2,3,-4)$ $B:(-5,9,-5)$ $C:(5,9,-1)$ | |
| | $AB^2=7^2+6^2+1^2=86$ | |
| | $AC^2=3^2+6^2+3^2=54$ | |
| | $BC^2=10^2+0^2+4^2=116$ Finding all three sides | M1 |
| | $\cos\angle ACB=\frac{116+54-86}{2\sqrt{116}\sqrt{54}}$ $(=0.53066\ ...)$ | M1 A1 |
| | $\angle ACB=57.95°$ awrt 57.95° | A1 **(4)** |
| | If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c). | |

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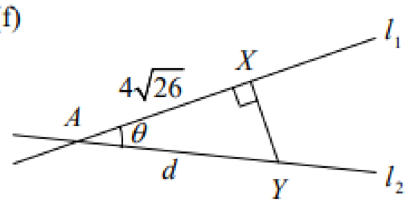
11.

| Question Number | Scheme | | Marks | |
|---|---|---|---|---|
| Q4 | (a) $A$: $(-6, 4, -1)$ | Accept vector forms | B1 | (1) |
| | (b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}3\\-4\\1\end{pmatrix} = 12+4+3 = \sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$ | | M1 A1 | |
| | $\cos\theta = \frac{19}{26}$ | awrt 0.73 | A1 | (3) |
| | (c) $X$: $(10, 0, 11)$ | Accept vector forms | B1 | (1) |
| | (d) $\overrightarrow{AX} = \begin{pmatrix}10\\0\\11\end{pmatrix} - \begin{pmatrix}-6\\4\\-1\end{pmatrix}$ | Either order | M1 | |
| | $= \begin{pmatrix}16\\-4\\12\end{pmatrix}$ | cao | A1 | (2) |
| | (e) $\lvert\overrightarrow{AX}\rvert = \sqrt{16^2+(-4)^2+12^2}$ | | M1 | |
| | $= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$ * | Do not penalise if consistent incorrect signs in (d) | A1 | (2) |
| | (f)  | Use of correct right angled triangle | M1 | |
| | | $\frac{\lvert\overrightarrow{AX}\rvert}{d} = \cos\theta$ | M1 | |
| | | $d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9 | A1 | (3) |
| | | | | [12] |