Differentiation- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

$$f(x) = 10e^{-0.25x} \sin x, \quad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

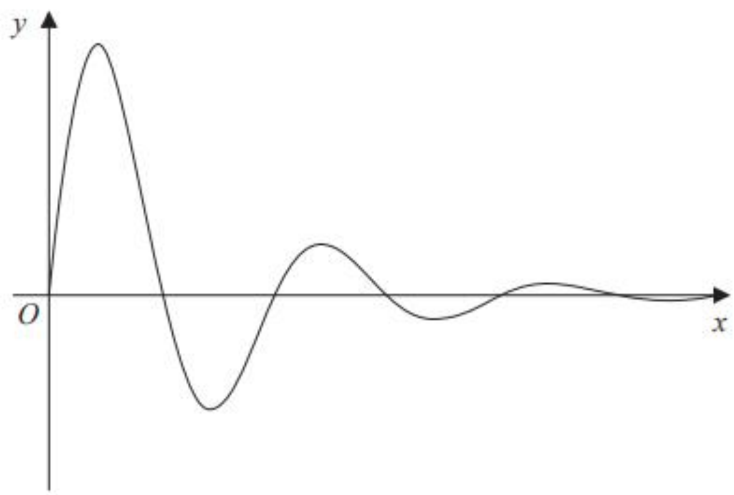

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = \left|10e^{-0.25t} \sin t\right| \quad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

2.

The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

3.

A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

4.

Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

5.

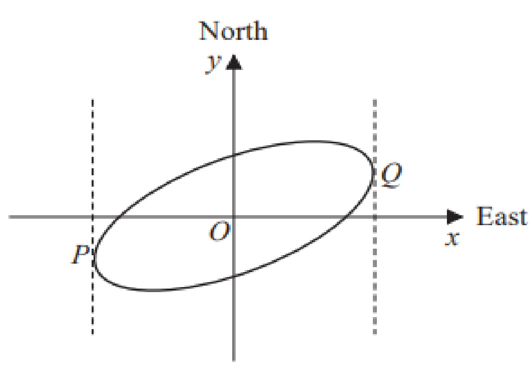

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

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6.

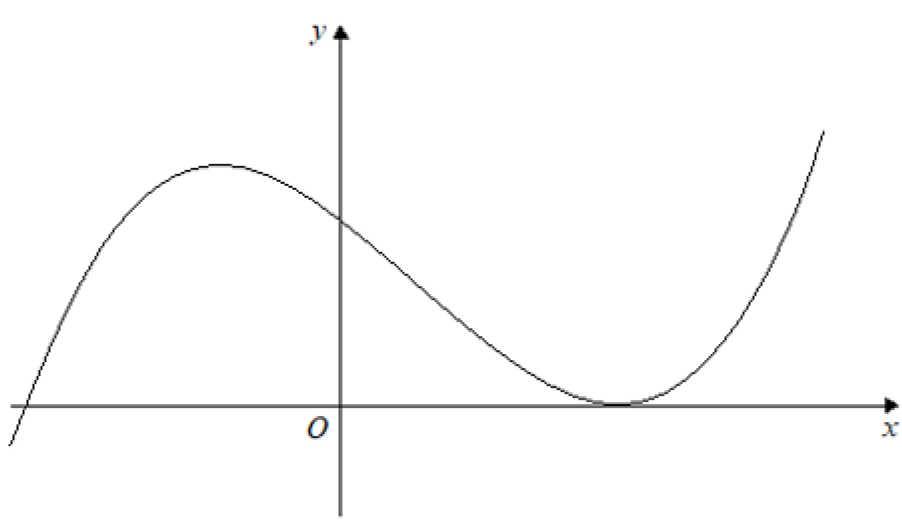

Figure 2

Figure 2 shows a sketch of part of the curve $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where

$$\mathrm{f}(x) = (2x - 5)^2 (x + 3)$$

(*a*) Given that

(i) the curve with equation $y = \mathrm{f}(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k,

(ii) the curve with equation $y = \mathrm{f}(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant c.

(3)

(*b*) Show that $\mathrm{f}'(x) = 12x^2 - 16x - 35$

(3)

Points A and B are distinct points that lie on the curve $y = \mathrm{f}(x)$.

The gradient of the curve at A is equal to the gradient of the curve at B.

Given that point A has x coordinate 3

(*c*) find the x coordinate of point B.

(5)

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7.

Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0,$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Give each term in your answer in its simplified form.

(6)

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8.

8.

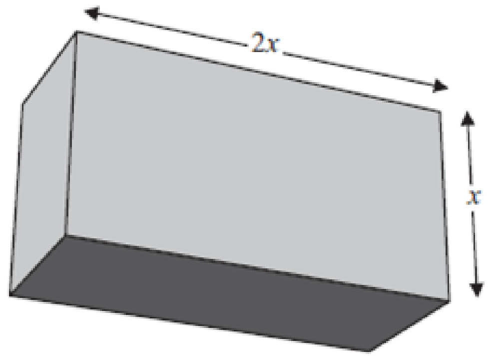

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}.$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

9.

7. (i) Given $y = 2x(x^2 - 1)^5$, show that

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{g}(x)(x^2 - 1)^4$ where $\mathrm{g}(x)$ is a function to be determined.

(4)

(*b*) Hence find the set of values of x for which $\frac{\mathrm{d}y}{\mathrm{d}x} \geqslant 0$

(2)

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x in its simplest form.

(4)

10.

8.

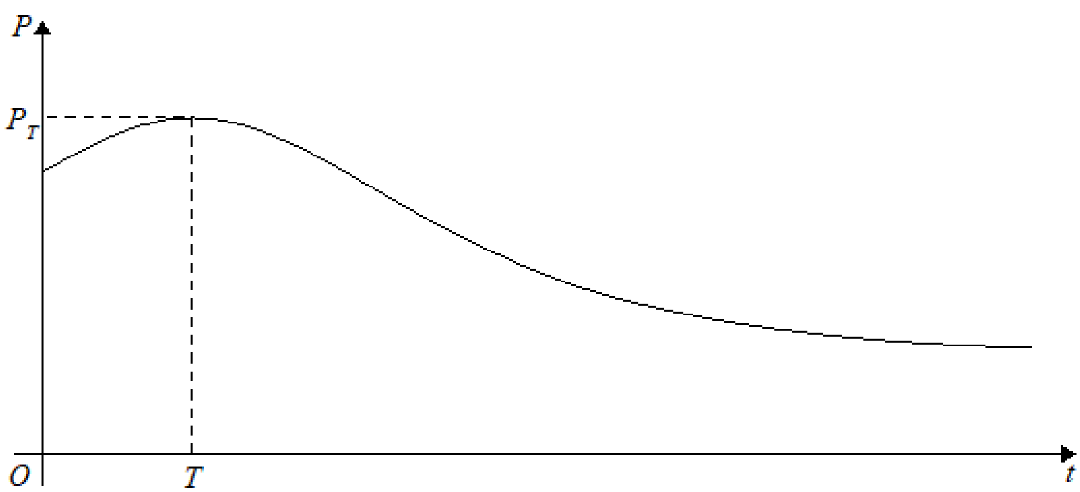

Figure 3

The number of rabbits on an island is modelled by the equation

$$P = \frac{100\mathrm{e}^{-0.1t}}{1+3\mathrm{e}^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 3.

(*a*) Calculate the number of rabbits that were introduced onto the island.

(1)

(*b*) Find $\dfrac{dP}{dt}$

(3)

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(*c*) Using your answer from part (*b*), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below k, where k is a positive constant.

(*d*) Use the model to state the maximum value of k.

(1)

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11.

2.

$$y = \frac{4x}{x^2 + 5}.$$

(*a*) Find $\dfrac{dy}{dx}$, writing your answer as a single fraction in its simplest form.

(4)

(*b*) Hence find the set of values of x for which $\dfrac{dy}{dx} < 0$.

(3)

12.

5. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leq x < \frac{\pi}{2}.$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2 2y$, $0 < y < \dfrac{\pi}{4}$, find $\dfrac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4},$$

where p and q are constants to be determined.

(5)

13.

6. $$\mathrm{f}(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \qquad x > 2, \quad x \in \mathbb{R}.$$

(*a*) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x-2},$$

find the values of the constants A and B.

(4)

(*b*) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = \mathrm{f}(x)$ at the point where $x = 3$.

(5)

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14.

5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(*a*) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(*b*) Use calculus to find the coordinates of A.

(6)

15.

9. Given that k is a **negative** constant and that the function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \qquad x \geq 0,$$

(*a*) show that $\mathrm{f}(x) = \dfrac{x+k}{x-2k}$.

(3)

(*b*) Hence find $\mathrm{f}'(x)$, giving your answer in its simplest form.

(3)

(*c*) State, with a reason, whether $\mathrm{f}(x)$ is an increasing or a decreasing function. Justify your answer.

(2)

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16.

1. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \qquad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

17.

3. The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

18.

8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}}, \qquad t \geq 0, \quad t \in □$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270.

(1)

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19.

5. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(*a*) find $\frac{dx}{dy}$ in terms of y.

(2)

(*b*) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(*c*) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

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20.

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(*a*) the value of w,

(2)

(*b*) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

21.

5. (i) Differentiate with respect to x

(*a*) $y = x^3 \ln 2x$,

(*b*) $y = (x + \sin 2x)^3$.

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$.

(5)

22.

7. $$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0.$$

(*a*) Show that $h(x) = \dfrac{2x}{x^2+5}$.

(4)

(*b*) Hence, or otherwise, find $h'(x)$ in its simplest form.

(3)

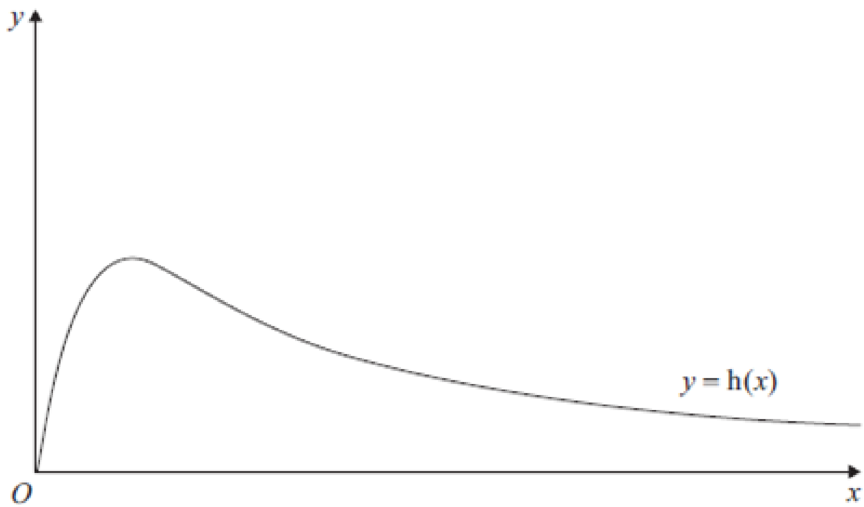

Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(*c*) Calculate the range of $h(x)$.

(5)

23.

8. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500.$$

where V is the value of the car in pounds (£) and t is the age in years.

(*a*) Find the value of the car when $t = 0$.

(1)

(*b*) Calculate the exact value of t when $V = 9500$.

(4)

(*c*) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)

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24.

3.

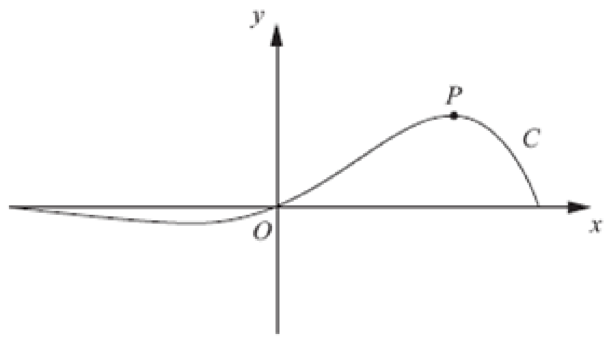

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}.$$

(*a*) Find the x-coordinate of the turning point P on C, for which $x > 0$. Give your answer as a multiple of π.

(6)

(*b*) Find an equation of the normal to C at the point where $x = 0$.

(3)

25.

7. (*a*) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln (3x)$,

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(*b*) Given that $x = 3 \tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

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26.

1. Differentiate with respect to x, giving your answer in its simplest form,

(*a*) $x^2 \ln (3x)$,

(4)

(*b*) $\dfrac{\sin 4x}{x^3}$.

(5)

27.

4. The point P is the point on the curve $x = 2\tan\left(y + \dfrac{\pi}{12}\right)$ with y-coordinate $\dfrac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

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28.

1. Differentiate with respect to x

(*a*) $\ln (x^2 + 3x + 5)$,

(2)

(*b*) $\dfrac{\cos x}{x^2}$.

(3)

29.

5. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt},$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(*a*) Write down the value of p.

(1)

(*b*) Show that $k = \dfrac{1}{4}\ln 3$.

(4)

(*c*) Find the value of t when $\dfrac{dm}{dt} = -0.6 \ln 3$.

(6)

30.

7. $$\mathrm{f}(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}.$$

(*a*) Show that

$$\mathrm{f}(x) = \frac{5}{(2x+1)(x+3)}.$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(*b*) Find an equation of the normal to C at P.

(8)

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31.

4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90 °C,

(*a*) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

(*b*) Show that $k = \frac{1}{5} \ln 2$.

(3)

(*c*) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

32.

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}.$$

(a) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

33.

8. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(a) show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y,$$

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

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34.

2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

35.

5.

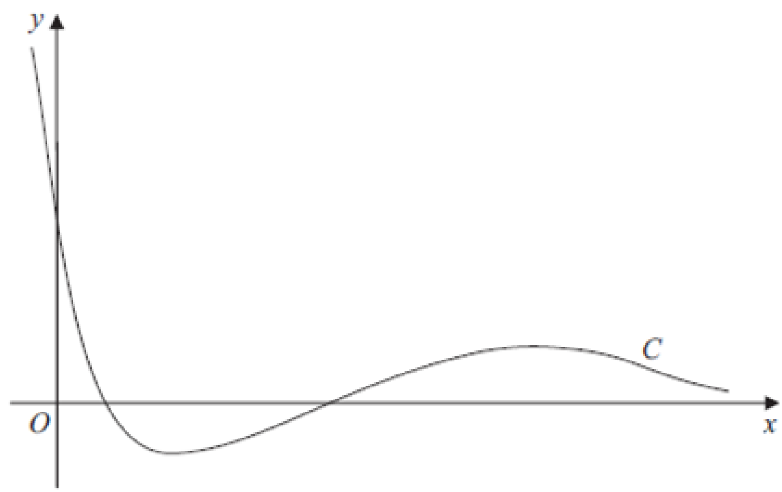

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\frac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)

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36.

4. (i) Given that $y = \dfrac{\ln(x^2+1)}{x}$, find $\dfrac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$.

(5)

37.

7. (*a*) By writing $\sec x$ as $\dfrac{1}{\cos x}$, show that $\dfrac{d(\sec x)}{dx} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(*b*) find $\dfrac{dy}{dx}$.

(4)

The curve with equation $y = e^{2x} \sec 3x$, $-\dfrac{\pi}{6} < x < \dfrac{\pi}{6}$, has a minimum turning point at (a, b).

(*c*) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

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38.

4. The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(*a*) Find the exact value of $\dfrac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(*b*) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

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39.

3. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17.$$

(*a*) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(*b*) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

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40.

2. The curve C has equation

$$x^2 - 3xy - 4y^2 + 64 = 0.$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(*b*) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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41.

1. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(*b*) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

42.

4.

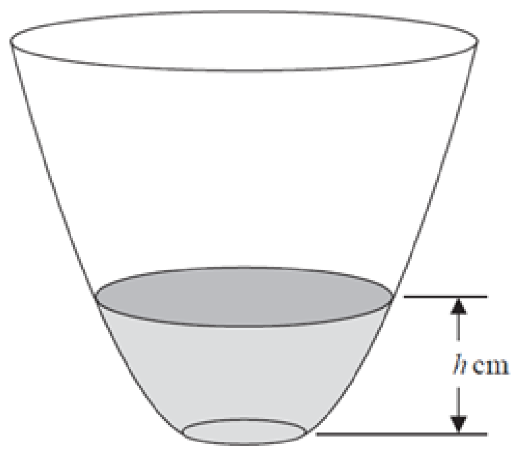

Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \qquad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π cm^3 s^{-1}.

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$.

(5)

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43.

7. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x-coordinate of Q is negative,

(*b*) use your answer to part (*a*) to find the coordinates of Q.

(7)

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44.

2.

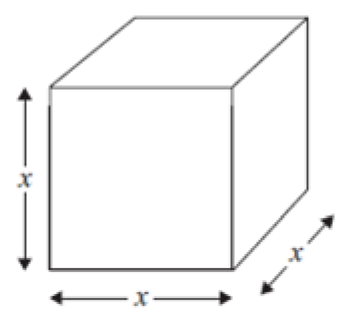

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm^3.

(a) Show that $\frac{dV}{dx} = 3x^2$.

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1},

(b) find $\frac{dx}{dt}$ when $x = 8$,

(2)

(c) find the rate of increase of the total surface area of the cube, in cm^2 s^{-1}, when $x = 8$.

(3)

45.

5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0.$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(7)

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46.

1. The curve C has the equation $2x + 3y^2 + 3x^2y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

47.

3.

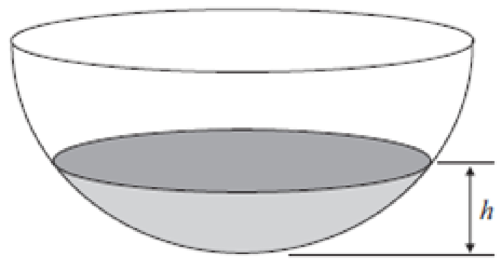

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.

When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \leq h \leq 0.25.$$

(*a*) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$.

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3 s^{-1}.

(*b*) Find the rate of change of h, in m s^{-1}, when $h = 0.1$.

(2)

48.

5. Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \qquad x > 0, \quad y > 0,$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

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49.

2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{dI}{dt}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(5)

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50.

3. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(7)

51.

8.

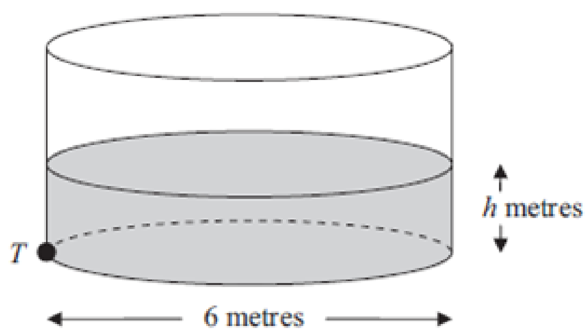

Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of 0.48π m^3 min^{-1}. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h$ m^3 min^{-1}.

(*a*) Show that, t minutes after the tap has been opened,

$$75\frac{dh}{dt} = (4 - 5h).$$

(5)

When $t = 0$, $h = 0.2$

(*b*) Find the value of t when $h = 0.5$

(6)

52.

3. The curve C has equation

$$\cos 2x + \cos 3y = 1, \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}.$$

(*a*) Find $\frac{dy}{dx}$ in terms of x and y.

(3)

The point P lies on C where $x = \frac{\pi}{6}$.

(*b*) Find the value of y at P.

(3)

(*c*) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers.

(3)

53.

6. The area A of a circle is increasing at a constant rate of 1.5 $cm^2\ s^{-1}$. Find, to 3 significant figures, the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2.

(5)