Trigonometric Functions - Questions

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

1.

4. (*a*) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants,

$R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(*b*) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(*c*) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

2.

3. (*a*) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of R and give the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(5)

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of θ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

(2)

3.

8. (*a*) Prove that

$$2\cot 2x + \tan x \equiv \cot x, \quad x \neq \frac{n\pi}{2}, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

June 2015 Mathematics Advanced Paper 1: Pure Mathematics 3

4.

1. Given that

$$\tan \theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1,$$

use standard trigonometric identities, to find in terms of p,

(*a*) $\tan 2\theta°$,

(2)

(*b*) $\cos \theta°$,

(2)

(*c*) $\cot (\theta - 45)°$.

(2)

Write each answer in its simplest form.

5.

3.

$$g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta.$$

Given that $g(\theta) = R \cos (2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the exact value of R and the value of α to 2 decimal places.

(3)

(*b*) Hence solve, for $-90° < \theta < 90°$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1,$$

giving your answers to one decimal place.

(5)

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(*c*) state the range of possible values of k.

(2)

6.

8. (*a*) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \quad n \in \mathbb{Z}.$$

(5)

(*b*) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}.$$

Give your answers to 3 decimal places.

(4)

June 2014 Mathematics Advanced Paper 1: Pure Mathematics 3

7.

7. (*a*) Show that

$$\text{cosec } 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in$$ □

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\text{cosec } (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

8.

9. (*a*) Express $2 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

$$H(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(*b*) (i) the maximum value of $H(\theta)$,

(ii) the smallest value of θ, for $0 \leq \theta \leq \pi$, at which this maximum value occurs.

(3)

Find

(*c*) (i) the minimum value of $H(\theta)$,

(ii) the largest value of θ, for $0 \leq \theta \leq \pi$, at which this minimum value occurs.

(3)

June 2013 Mathematics Advanced Paper 1: Pure Mathematics 3

9.

8.

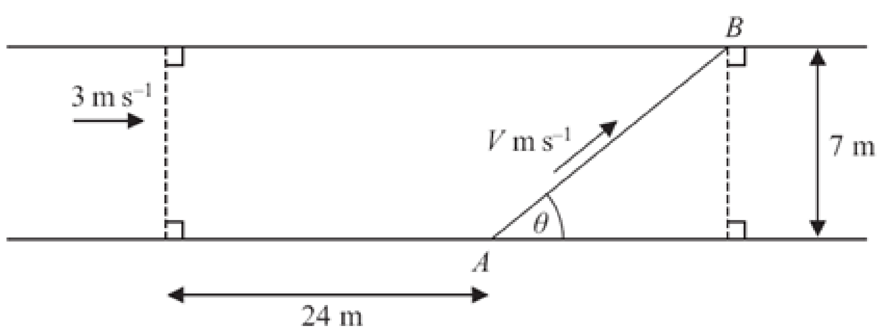

Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1}.

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A.
John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2.

Kate's speed is V m s^{-1} and she moves in a straight line, which makes an angle θ, $0 < \theta < 150°$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \qquad 0 < \theta < 150°$$

(*a*) Express $24\sin \theta + 7\cos \theta$ in the form $R \cos (\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

(*b*) find the minimum value of V.

(2)

Given that Kate's speed has the value found in part (*b*),

(*c*) find the distance AB.

(3)

Given instead that Kate's speed is 1.68 m s^{-1},

(*d*) find the two possible values of the angle θ, given that $0 < \theta < 150°$.

(6)

Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 3

10.

4. (*a*) Express $6 \cos \theta + 8 \sin \theta$ in the form $R \cos (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*)
$$\mathrm{p}(\theta) = \frac{4}{12 + 6 \cos \theta + 8 \sin \theta}, \qquad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

June 2012 Mathematics Advanced Paper 1: Pure Mathematics 3

11.

5. (*a*) Express $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin\theta$ and $\cos\theta$.

(2)

(*b*) Hence show that

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta .$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π.

(3)

12.

8.

$$f(x) = 7\cos 2x - 24\sin 2x.$$

Given that $f(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α.

(3)

(*b*) Hence solve the equation

$$7\cos 2x - 24\sin 2x = 12.5$$

for $0 \le x < 180°$, giving your answers to 1 decimal place.

(5)

(*c*) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found.

(2)

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14\cos^2 x - 48\sin x\cos x.$$

(2)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 3

13.

5. Solve, for $0 \le \theta \le 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5.$$

Give your answers in degrees to 1 decimal place.

(10)

June 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

14.

6. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1.$$

(5)

15.

8. (*a*) Express $2 \cos 3x - 3 \sin 3x$ in the form $R \cos (3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x} \cos 3x.$$

(*b*) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x} \cos (3x + \alpha),$$

where R and α are the constants found in part (*a*).

(5)

(*c*) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 3

16.

1. (*a*) Express $7\cos x - 24\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

(*b*) Hence write down the minimum value of $7\cos x - 24\sin x$.

(1)

(*c*) Solve, for $0 \leq x < 2\pi$, the equation

$$7\cos x - 24\sin x = 10,$$

giving your answers to 2 decimal places.

(5)

June 2010 Mathematics Advanced Paper 1: Pure Mathematics 3

17.

7. (*a*) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2\sin\theta - 1.5\cos\theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

18.

8. Solve

$$\text{cosec}^2\, 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(7)