Vectors - Questions

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 4

1.

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point X.

(*a*) Find the coordinates of the point X.

(3)

(*b*) Find the size of the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point A lies on l_1 and has position vector $\begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$

(*c*) Find the distance AX, giving your answer as a surd in its simplest form.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YA}$ is perpendicular to the line l_1

(*d*) find the distance YA, giving your answer to one decimal place.

(2)

The point B lies on l_1 where $|\overrightarrow{AX}| = 2|\overrightarrow{AB}|$.

(*e*) Find the two possible position vectors of B.

(3)

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2.

8. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix},$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$.

(*a*) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$.

The line l_2 passes through the point P and is parallel to the line l_1.

(*b*) Write down a vector equation for the line l_2.

(2)

(*c*) Find the exact value of the distance AP.
Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(2)

The acute angle between AP and l_2 is θ.

(*d*) Find the value of $\cos\theta$.

(3)

A point E lies on the line l_2.
Given that $AP = PE$,

(*e*) find the area of triangle APE,

(2)

(*f*) find the coordinates of the two possible positions of E.

(5)

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3.

4. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix},$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(*a*) Find the coordinates of A.
(2)

(*b*) Find the value of the constant p.
(3)

(*c*) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.
(3)

The point B lies on l_2 where $\mu = 1$.

(*d*) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures.
(3)

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4.

8. Relative to a fixed origin O, the point A has position vector $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point B has position vector $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$.

The line l_1 passes through the points A and B.

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(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Hence find a vector equation for the line l_1.

(1)

The point P has position vector $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$.

Given that angle PBA is θ,

(c) show that $\cos\theta = \frac{1}{3}$.

(3)

The line l_2 passes through the point P and is parallel to the line l_1.

(d) Find a vector equation for the line l_2.

(2)

The points C and D both lie on the line l_2.

Given that $AB = PC = DP$ and the x coordinate of C is positive,

(e) find the coordinates of C and the coordinates of D.

(3)

(f) find the exact area of the trapezium $ABCD$, giving your answer as a simplified surd.

(4)

June 2013 Mathematics Advanced Paper 1: Pure Mathematics 4

5.

8. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(*a*) find the value of p.

(4)

Given also that B is a point on l such that $\angle BPA = 45°$,

(*b*) find the coordinates of the two possible positions of B.

(5)

June 2012 Mathematics Advanced Paper 1: Pure Mathematics 4

6.

8. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(*c*) find the position vector of the point P.

(6)

7.

7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,

the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,

and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

(c) Show that the size of the angle BAD is 109°, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.

(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)

June 2011 Mathematics Advanced Paper 1: Pure Mathematics 4

8.

6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \qquad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where μ and λ are scalar parameters.

(*a*) Show that l_1 and l_2 meet and find the position vector of their point of intersection A.
(6)

(*b*) Find, to the nearest $0.1°$, the acute angle between l_1 and l_2.
(3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(*c*) Show that B lies on l_1.
(1)

(*d*) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.
(4)

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9.

4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(*a*) Find $\overrightarrow{AB}$.
(2)

(*b*) Find a vector equation of l.
(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant.

Given that AC is perpendicular to l, find

(*c*) the value of p,
(4)

(*d*) the distance AC.
(2)

June 2010 Mathematics Advanced Paper 1: Pure Mathematics 4

10.

7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(*a*) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(*b*) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(*c*) Hence, or otherwise, find the area of the triangle ABC.

(5)

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11.

4. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(*a*) Write down the coordinates of A.

(1)

(*b*) Find the value of $\cos\theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X.

(1)

(d) Find the vector $\overrightarrow{AX}$.

(2)

(e) Hence, or otherwise, show that $\left|\overrightarrow{AX}\right| = 4\sqrt{26}$.

(2)

The point Y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(f) find the length of AY, giving your answer to 3 significant figures.

(3)